Solved Problems of Mechanics
Chapter-6 Friction

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Problem 6.1: Find out the maximum acceleration with which the man can move vertically upward such that the block does not slip on the ground. Mass of the block is m, Mass of the man is $2m$, Coefficient of friction between the block and the ground is 1/3.

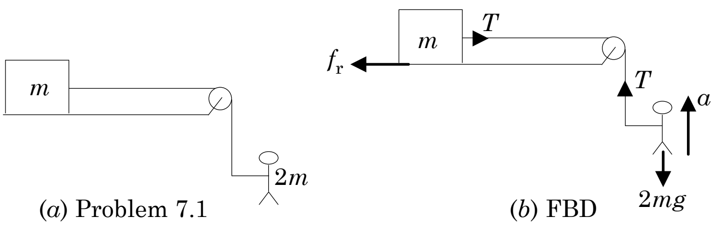


Diagram 6.1 Problem 6.1

Solution: Refer Diagram 6.1b, force balance equation,

$$T_{\max} = f_{r_{\max}} = \frac{1}{3}mg$$

and,
$$T_{\max} - 2mg = 2ma$$

or,
$$\frac{1}{3}mg - 2mg = 2ma$$

or,
$$a = -5g/6.$$

The maximum acceleration which the man cannot have in the downward direction is equal to g. the maximum force which the man can apply on the string $mg/3$.

Problem 6.2: A block is placed on wedge. If no relative motion takes place between the block and the wedge then find out the friction acting on the wedge from the ground. Mass of the block is & mass of the wedge is M, coefficient of friction between the wedge and the ground is k.

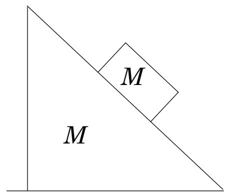


Diagram 6.2 Problem 6.2

Solution: Refer Diagram 6.2.
If the block does not slide on the wedge then the wedge will have no tendency of moving on the ground friction force on the wedge will be equal to zero.

Problem 6.3: In the diagram shown, find out the range of the value of force F such that no relative motion takes place between the blocks. Coefficient friction between the blocks is 1/2. No friction between the 3M mass and ground.

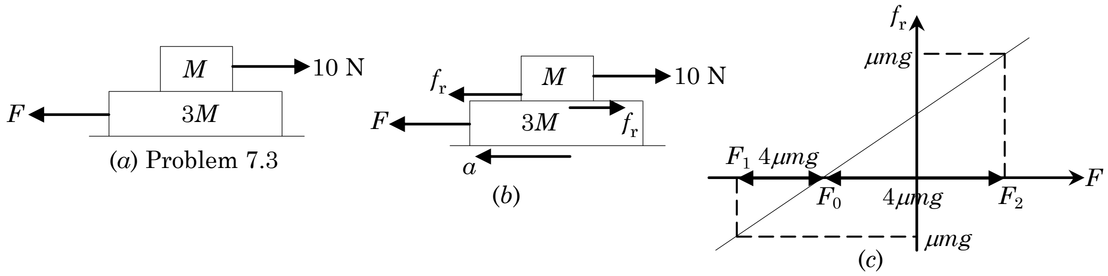


Diagram 6.3 Problem 6.3

Solution: Refer Diagram 6.3b.

Acceleration is $$a = \frac{F-10}{4M}$$

Consider upper block, $$f_r - 10 = Ma$$

or, $$f_r = 10 + Ma$$

or, $$f_r = 10 + \frac{F-10}{4} = \frac{F}{4} + \frac{30}{4}$$

If the force F has any value from F_1 to F_2 then the magnitude of friction less than equal to μMg, no relative motion will takes place.
When $F = -30$ N, then friction is zero, why?
Answer of the above question when $F = -30$ N, then the independent acceleration of both the blocks is same.

Thus, $$-\mu Mg = \frac{F_1 + 30}{4}$$

Therefore, $$F_1 = -4\mu Mg - 30$$

and, $$\mu Mg = \frac{F_2 + 30}{4}$$

Therefore, $$F_2 = 4\mu Mg - 30$$

Problem 6.4: The coefficient of friction at all the contact surfaces is k. Find out the minimum force that should be applied on the block so that it just starts moving.

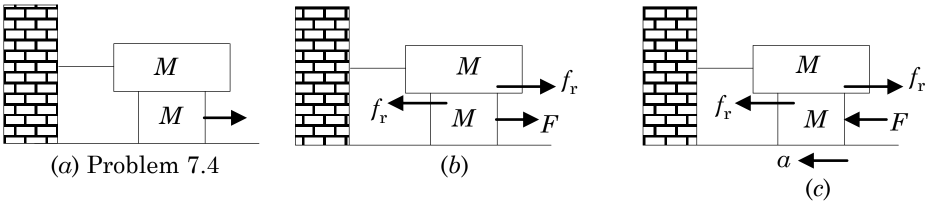


Diagram 6.4 Problem 6.4

Solution: Refer Diagram 6.4b. If F is applied in the forward direction, the lower will move with respect to upper block for $F \geq \mu Mg$.

Refer Diagram 6.4c, $f_r = Ma$, and $f_r = F/2$.

so, $f_r = \mu Mg$

Thus, $F = 2\mu Mg$

In this question we cannot applied the graphical technique because in the first part only one block moving and in the second part both the blocks are moving.

Problem 6.5: Repeat the analysis of Problem 6.4. Find out the minimum value of F so that relative motion just starts. Consider the horizontal ground to be smooth.

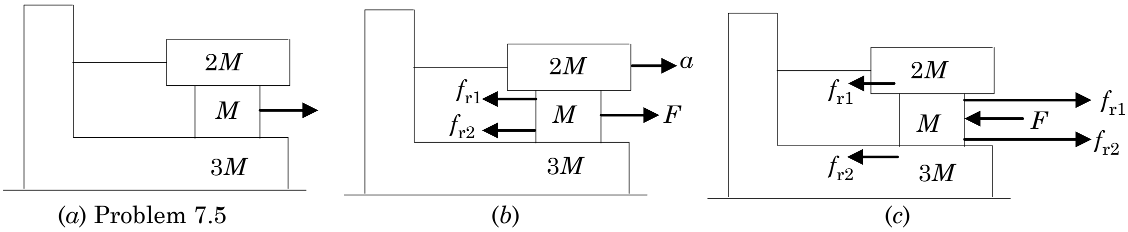


Diagram 6.5 Problem 6.5

Solution: Refer Diagram 6.5b. Relative motion will start between 2M, and M, and between M and 3M at the same time.

Acceleration is $a = F/6M$

$$f_{r_1} = k2Mg, \text{ and } f_{r_2} = k3Mg$$

Thus, $F = f_{r_1} - f_{r_2} = Ma$

or, $$F = 2kMg - 3kMg = M \times \frac{F}{6M}$$

or, $F = 6kMg$.

This is the minimum value of force F.

Refer Diagram 6.5c, here the direction of acceleration is reverse, thus

$$f_{r_1} = 2Ma = 2F/6 = F/3,$$

Therefore, $F/3 = 2kMg$, or, $F_1 = 6kMg$.

and $f_{r_2} = 3Ma = 3F/6 = F/2$

Therefore, $F/2 = 3kMg$, or, $F_2 = 6kMg$.

Problem 6.6: A man is trying to pull a block as shown. Find out the condition for the man to successfully pull the block. Mass of man M and mass of the block is m. The string is inclined at an angle α with the horizontal.

Solution: the man can move the block only, if $(f_{r_2})_{\max}$ is greater than $(f_{r_1})_{\max}$

Refer Diagram 6.6b,

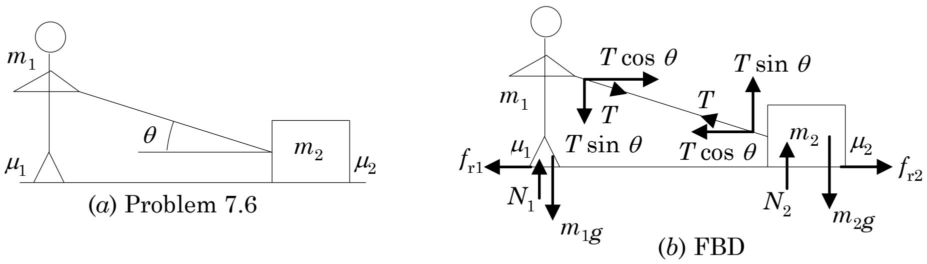


Diagram 6.6 Problem 6.6

$N_1 = m_1g + T\sin\theta$

1. $\mu_1 N_1 > T\cos\theta$

$N_2 = m_2g - T\sin\theta$

2. $T\cos\theta > \mu_2 N_2$

Thus, $\mu_1(m_1g + T\sin\theta) > T\cos\theta$

or, $T < \dfrac{\mu_1 m_1 g}{\cos\theta - \mu_1\sin\theta}$

and, $T\cos\theta > \mu_2(m_2g - T\sin\theta)$

or, $T > \dfrac{\mu_2 m_2 g}{\cos\theta + \mu_2\sin\theta}$

Therefore, $\dfrac{\mu_1 m_1 g}{\cos\theta - \mu_1\sin\theta} > \dfrac{\mu_2 m_2 g}{\cos\theta + \mu_2\sin\theta}$

Problem 6.7: A wedge of mass M is placed on a smooth xz plane as shown. On rough inclined surface of the wedge, a block of mass m is placed. By applying a force F in the $-z$ direction it is desired to just start relative motion between the wedge and the block. Find the minimum value of F. The coefficient of friction between the wedge and the block is $2\tan\theta$.

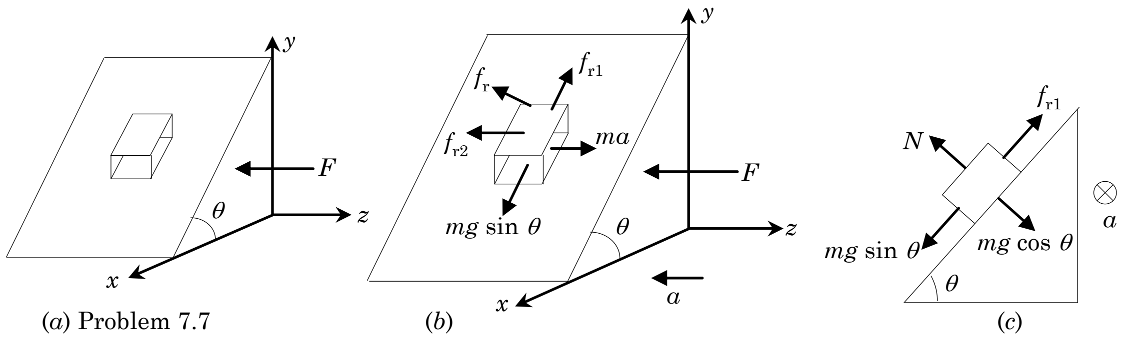


Diagram 6.7 Problem 6.7

Solution: Refer Diagram 6.7b. The friction force f_r will have two components normal reaction will remains equal to $mg\cos\theta$. Angle between pshudo force and the normal reaction will remains equal to 90°.

$$f_{r_1} = mg\sin\theta, \text{ and } f_{r_2} = ma$$

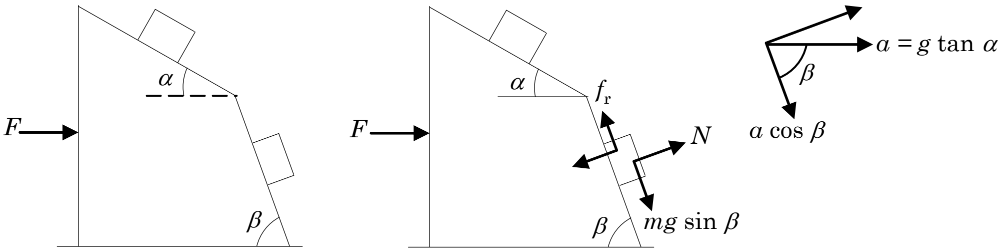
or, $f_r = \sqrt{f_{r_1}^2 + f_{r_2}^2}$

or, $f_r = \mu N.$

Thus, $2\tan\theta \cdot mg\cos\theta = \sqrt{(mg\sin\theta)^2 + (ma)^2}$

or, $a = \sqrt{3}g\sin\theta$

Problem 6.8: In the diagram shown, the upper block has no tendency of relative motion with respect to wedge. What will be the direction of friction acting on the lower block? Assume the lower block to remain in the state of rest with respect to the wedge. Find out the magnitude of friction force acting on the lower block. Both the blocks are of the mass M.



(*a*) Problem 7.8 (*b*)

Diagram 6.8 Problem 6.8

Solution: Refer Diagram 6.8b, force balance equation,

$$mg \sin \beta - f_r = ma \cos \beta$$

We not need find the normal reaction, if we want to solve the problem.

Problem 6.9: Find out the coefficient of friction between the M and the $2M$ such no relative motion can take place between them. Assume no friction to be present at any other surface.

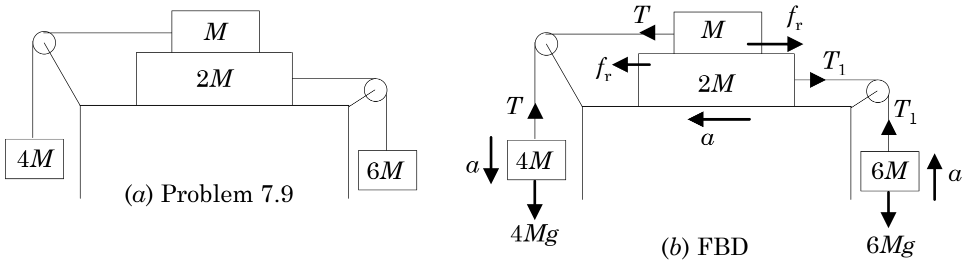


Diagram 6.9 Problem 6.9

Solution: Refer Diagram 6.9b, force balance equation,

$$4Mg - T = 4Ma \qquad [i]$$

$$T - T_1 = 3Ma \qquad [ii]$$

$$T_1 - 6Mg = 6Ma \qquad [iii]$$

On solving Eqns. (*i*), (*ii*), and (*iii*), so we have

$$a = -2g/13$$

Consider upper block, and lower block,

$$T - f_r = Ma \qquad [iv]$$

$$4Mg - T = 4Ma \qquad [v]$$

On solving Eqns. (*iv*), and (*v*), so we have

$$f_r = 4Mg - 5Ma$$

or,
$$f_r = 4Mg + \frac{10}{13}Mg = \frac{62}{13}Mg$$

thus,
$$\mu Mg \geq \frac{62}{13}Mg$$

or,
$$\mu \geq 62/13.$$

Problem 6.10: No friction is present between the block and the wedge. The coefficient of friction between the wedge and the ground is 1/2. If the wedge is of mass M then find out the minimum value of the mass of the block such relative motion can just start between the wedge and the ground. No friction anywhere else. Angle of inclination of the wedge is θ. Mass of the wedge is M.

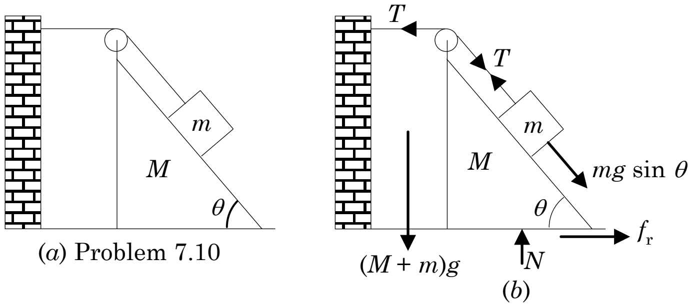


Diagram 6.10 Problem 6.10

Solution: Refer Diagram 6.10b, force balance equation

$$T = mg \sin \theta \qquad [i]$$

$$T = f_r \qquad [ii]$$

On solving Eqns. (i), and (ii), so we have

$$f_r = mg \sin \theta$$

or,
$$\mu N \geq mg \sin \theta$$

or,
$$(M + m)g \geq mg \sin \theta$$

or,
$$\frac{\mu M}{\sin \theta - \mu} \geq m$$

In above equation if we change the inequality then relative motion will be started.
If $\mu = \sin \theta$, then we get the result that for any value of m no relative motion will start between the block and wedge.

Problem 6.11: Find out the minimum value of friction between the smaller block and the larger block such that no relative motion can take place between them. The force applied by the man is $2Mg$. Man remains in the state of rest with respect to the bigger block.

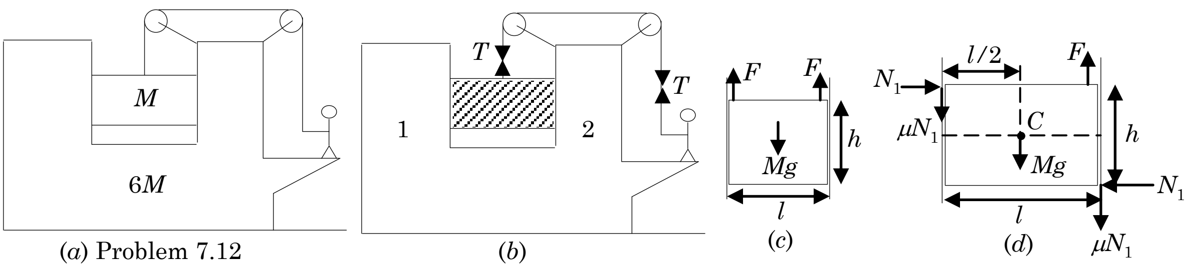


Diagram 6.11 Problem 6.11

Solution: Refer Diagram 6.11b, the shaded block is moving vertically upward. The bigger block will remain in the state of rest. At the surface 1 and 2, no normal reaction will be present. Even if these surfaces are rough no friction force can act.
Consider Diagram 6.11c, in the initial state the plate is moving vertically upward with constant velocity.

If we remove one force F [diagram 6.11d] then what should be the value of force acting on the other hand so that plate can move upward. Coefficient of friction between the plate and the wall is μ.

Thus, $$F \geq Mg + 2\mu N_1$$

Taken moment about point C,

$$\mu N_1 \frac{l}{2} - \mu N_1 \frac{l}{2} - N_1 \frac{h}{2} + F \frac{l}{2} - N_1 \frac{h}{2} = 0$$

or, $$N_1 = \frac{F}{h} \frac{l}{2}$$

Therefore, $$F \geq Mg + \frac{2\mu l F}{2h}$$

or, $$F \geq \frac{Mg}{\frac{1 - 2\mu l}{2h}}$$

If $\mu = h/l$, then there is no value of force F which can pull the plate outside.

Problem 6.12: The coefficient of static friction between the blocks is 0.2 and coefficient of kinetic friction is 0.18. Both the blocks have the same mass 1 kg. A force increasing linearly with respect to time according to the relation $F = 2t$, has been applied. Draw the acceleration versus time graph of both the blocks.

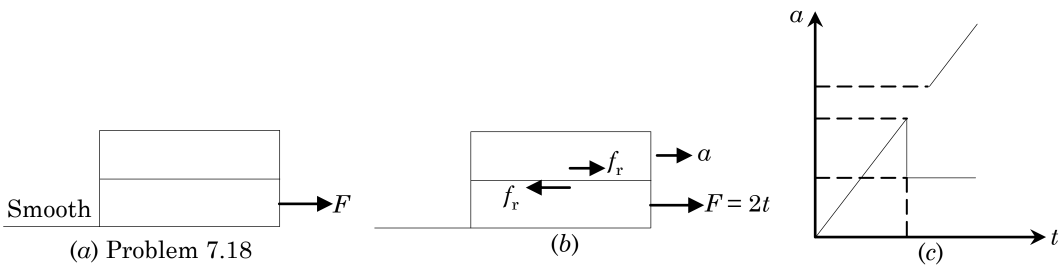


Diagram 6.12 Problem 6.12

Solution: Given that: $\mu_s = 0.2$, $\mu_k = 0.18$. Refer Diagram 6.12b,

$$f_r = 1 \text{ kg} \times a, \text{ and } f_r = \frac{F}{2.1 \text{ kg}} \times 1 \text{ kg} = \frac{F}{2}$$

When the relative motion is just about to start then the static friction on force will be acting on its maximum value.

So, $$F = 2\mu_s mg$$

Acceleration $$a = F/2m$$

Thus, $$\mu_s g = F/2$$

or, $$\mu_s g = 2 \text{ m/s}^2.$$

When $a = 2$ m/s^2, then relative motion about to start between two blocks.

When the relative motion just starts the friction force acting to block become minimum.

$$f_r = 1.8m, \quad \text{or} \quad a_1 = 1.8/1 \text{ m/s}^2.$$

The value of the force F, then started the relative motion was $2\mu_s g$.

Thus, $$a_2 = \frac{F - f_r}{1} = \frac{4 - 1.8}{1} = 2.2 \text{ m/s}^2.$$