Solved Problems of Mechanics
Chapter-5 Force Concept

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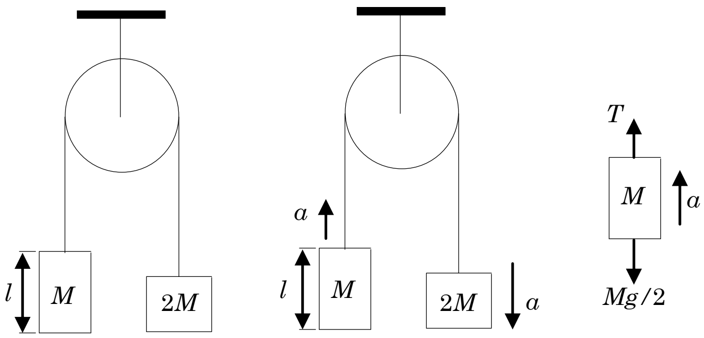
Please welcome for any correction or misprint in the entire manuscript and your valuable suggestions kindly mail us brijrbedu@gmail.com.

Problem 5.1: In the Diagram 5.1a shown, the uniform string of length l has mass M. The block is of mass $2M$. The whole system is released from the state of rest. Find out the tension at the midpoint of this string. The other string is massless.



(a) Problem (b) Free body diagram

Diagram 5.1 Problem 5.1

Solution: Refer to Diagram 5.1b,

Now, acceleration,
$$a = \frac{(2M - M)\,g}{2M + M} = \frac{g}{3}$$

Force balance equation
$$T - \frac{Mg}{2} = \frac{Ma}{2}$$

or,
$$T - \frac{Mg}{2} = \frac{M}{2}\left(g + \frac{g}{3}\right)$$

or,
$$T = 2Mg/3.$$

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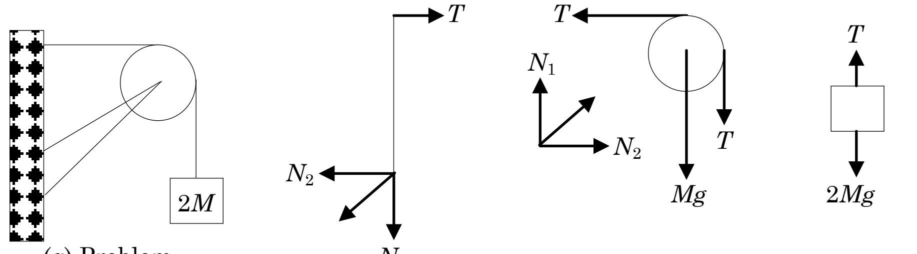
Problem 5.2: In the Diagram 5.2a shown, the axle + pulley has mass M and the block is of mass $2M$. Find out the total force acting on the wall due to the mechanical system shown. The string is massless.



(*a*) Problem (*b*) Free body diagram

Diagram 5.2 Problem 5.2

Solution: Refer to Diagram 5.2b. The total weight of the system is equal to $3Mg$. this weight is balanced by the force from the wall.
Force balance equation,

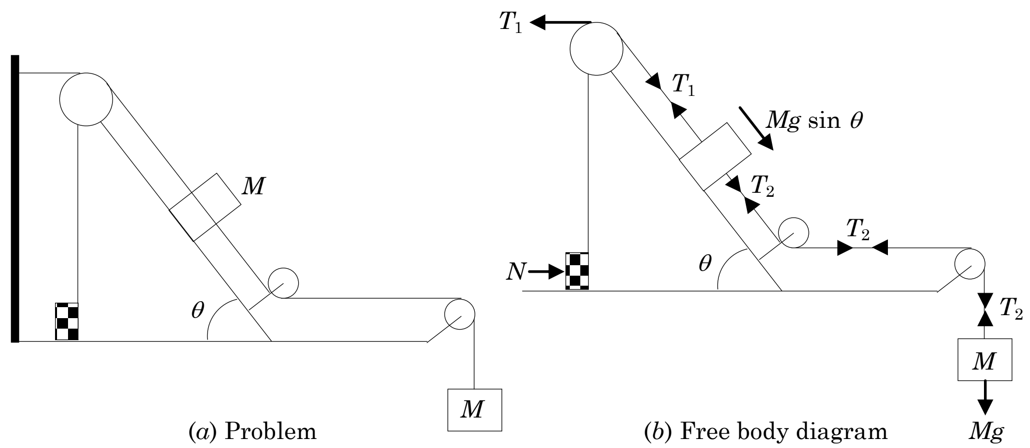
$$T = 2Mg \qquad [i]$$

$$T = N_2 \qquad [ii]$$

$$N_1 = T + Mg \qquad [iii]$$

Using Eqns. (*i*), (*ii*) and (*iii*), $N_1 = 3Mg$.
There are two forces acting on the pulley from the wall, one is tension by the help of string, and other is normal reaction by the help of axle.

Problem 5.3: In the Diagram 5.3a shown, both the blocks have the same mass M. The wedge has been held in the state of rest by the support. What is the force acting on the wedge from the support?



(*a*) Problem (*b*) Free body diagram

Diagram 5.3 Problem 5.3

Solution: Refer to Diagram 5.3b. In this problem important points are:

- The block and wedge have to remain in the state of rest.
- For obtaining the normal reaction N, we have to find out the $T_1 - T_2$, only.

Force balance equation,

$$T_2 = Mg \qquad [i]$$

$$T_1 = Mg \sin\theta + T_2 \qquad [ii]$$

or,

$$N = T_1 - T_2 = Mg \sin\theta$$

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Problem 5.4: If the force applied on the wedge is $2(M + m)g \tan \theta$ then find out the (compressional) tension in the rod. Mass of the wedge is M and mass of the block is m.

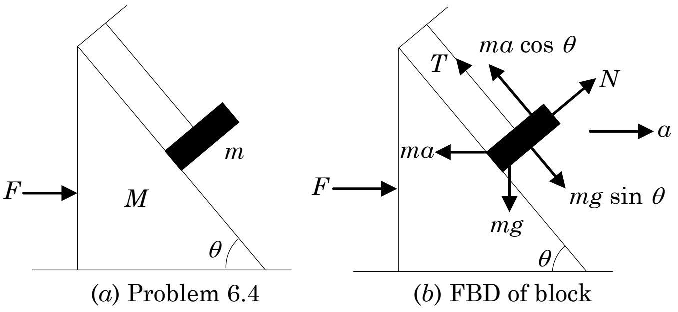

Diagram 5.4 Problem 5.4

Solution: Refer to Diagram 5.4b.

Acceleration, $$a = \frac{F}{M+m} = \frac{2(M+m)g\tan\theta}{M+m} = 2g\tan\theta$$

Consider free body diagram of the block with respect to wedge,

$$T + ma\cos\theta = mg\sin\theta$$

or, $$T + ma\,(2g\tan\theta)\cos\theta = mg\sin\theta$$

or, $$T = -\,mg\sin\theta$$

The tension present in rod will be compressive because it is –ve.

Problem 5.5: In the Diagram 5.5a shown, the blocks have the masses as mentioned on them. If the string is pulled with a force F then find out the acceleration of the $6M$ mass.

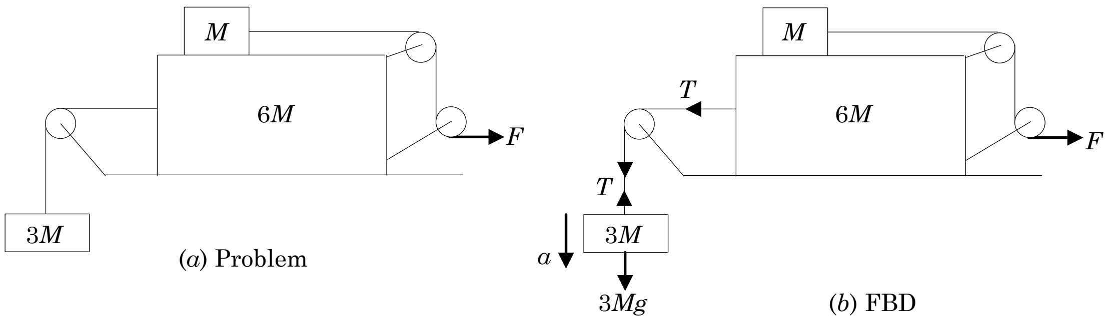

Diagram 5.5 Problem 5.5

Solution: Refer to Diagram 5.5b,

Force balance equation

$$T = 6Ma \qquad [i]$$

$$3Mg - T = 3Ma \qquad [ii]$$

On solving Eqns. (i), and (ii), $a = g/3$.

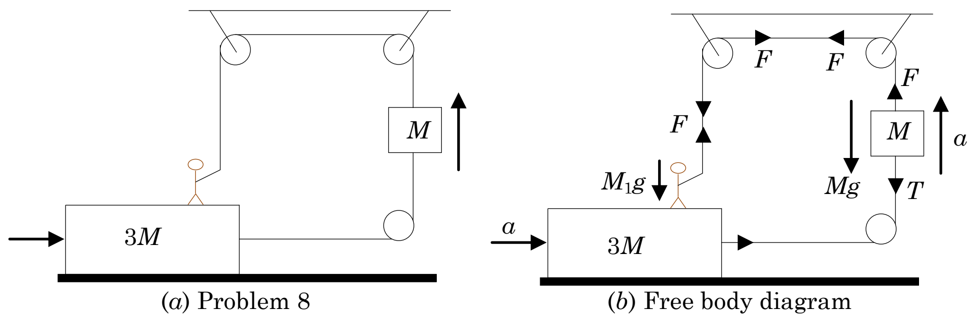

Diagram 5.6 Problem 5.6

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Problem 5.6: In the Diagram 5.6a shown, find out the minimum mass of the man so that he may keep himself in the state of rest with respect to earth and pull the two blocks with acceleration $2g$. What should be the force applied by the man on the string to achieve the desired acceleration?

Solution: Refer to Diagram 5.6b, force balance equation,

$$T = 3Ma \quad [i]$$

$$F - Mg - T = Ma \quad [ii]$$

On solving Eqns. (i), and (ii),

$$F - Mg = 4Ma$$

According to problem $a = 2g$, thus,

$$F = 9Mg$$

If we consider the weighing machine (M_1), then,

$$N + F = M_1g$$

or, $$N = M_1g - F$$

Therefore, $$M_1g \geq F$$

or, $$M_1 = 9F$$

reading on the machine will depend only on the normal reaction, and reading is equal N/g.

Problem 5.7: In the Diagram 5.7a shown, the wedge is sliding on the smooth inclined plane. Find out the direction of acceleration and the magnitude of acceleration of the block placed on the wedge with respect to earth.

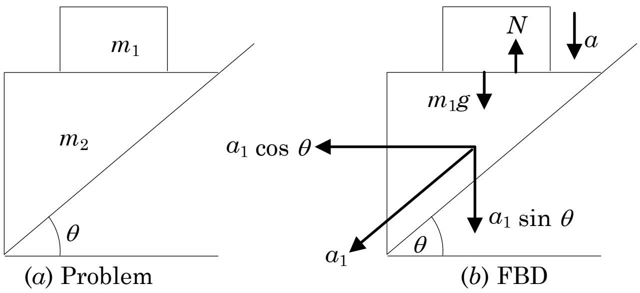

Diagram 5.7 Problem 5.7

Solution: Refer to Diagram 5.7b, with respect to earth the block will move in the vertical direction only and the wedge will move along the inclined plane only.
Now, form force balance equation,

$$a = a_1 \sin\theta$$

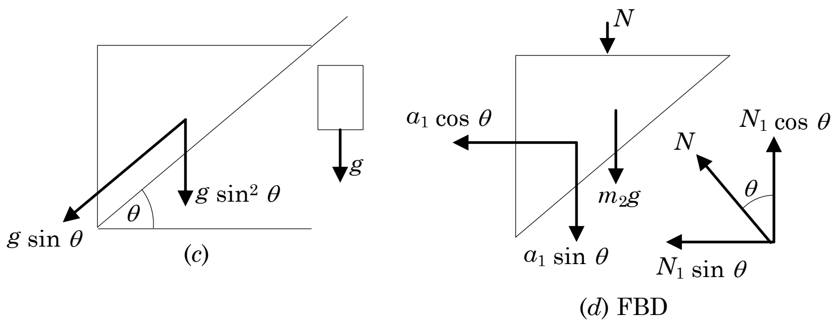

Diagram 5.8 Problem 5.7

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The independent vertical acceleration of block is greater than that of the wedge, when we place a block on the wedge, then block tends to move the wedge with the help of normal reaction.

Equations of block:

$$m_1g - N = m_1a \qquad [i]$$

Equations of wedge:

$$m_2g + N - N_1 \cos\theta = m_2a_1 \sin\theta \qquad [ii]$$

$$N_1 \sin\theta = m_2a_1 \cos\theta \qquad [iii]$$

Adding Eqns. (*i*) and (*ii*),

$$m_1g + m_2g - N_1 \cos\theta = (m_1a + m_2)\, a_1 \sin\theta \qquad [iv]$$

Using Eqns. (*iii*) and (*iv*),

or, $$(m_1 + m_2)\, g - m_2a_1 \frac{\cos^2\theta}{\sin\theta} = (m_1 + m_2)\, a_1 \sin\theta$$

or, $$a_1 = \frac{(m_1 + m_2)\, g \sin\theta}{m_1 \sin^2\theta + m_2}$$

let us derive above expression with conditions. It is less than g and grater than g sin θ, thus

$$m_1 + m_2 > m_1 \sin^2\theta + m_2$$

or, $$\frac{m_1 + m_2\; g \sin\theta}{m_1 \sin^2\theta + m_2} > 1$$

or, $$a_1 \sin\theta = \frac{(m_1 + m_2)\, g \sin^2\theta}{m_1 \sin^2\theta + m_2} > g \sin^2\theta$$

divided by $\sin^2\theta$,

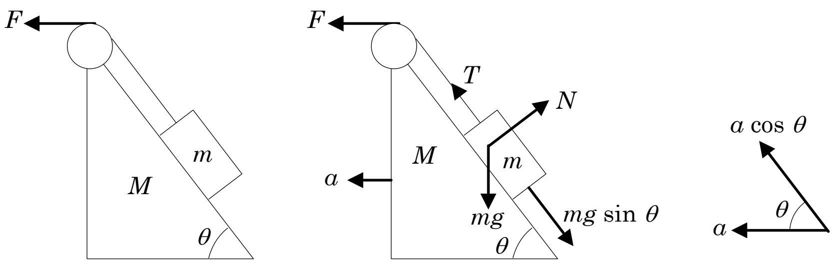
or, $$\frac{(m_1 + m_2)\, g}{m_1 + (m_2 / \sin^2\theta)} > g$$

or, $$\frac{m_1 + m_2}{m_1 + (m_2 / \sin^2\theta)} < 1$$

therefore

$$g > a_1 \sin\theta = \frac{(m_1 + m_2)\, g \sin^2\theta}{m_1 \sin^2\theta + m_2} > g \sin^2\theta$$

Problem 5.8: In the Diagram 5.9a shown, if no relative motion takes place between the block and the wedge then find out the tension in the string.



(*a*) Problem 10 (*b*) Free body diagram

Diagram 5.9 Problem 5.8

Solution: Refer to Diagram 5.9b,

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Acceleration is $$a = \frac{F}{M+m}$$

force balance equation,

$$T - mg\sin\theta = ma\cos\theta$$

or, $$F = mg\sin\theta + \frac{mF}{M+m}\cos\theta$$

or, $$F = \frac{mg\sin\theta}{1 - \frac{m\cos\theta}{M+m}}$$

the force F will always +ve, because the denominator will be +ve.

Problem 5.9: Find out the acceleration of the smaller mass immediately after the system is released from the state of rest.

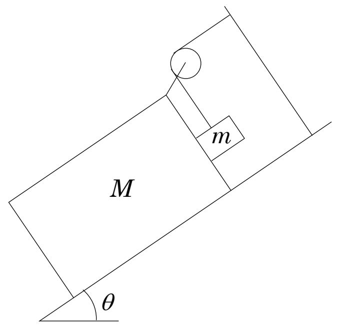

Diagram 5.10 Problem 5.9

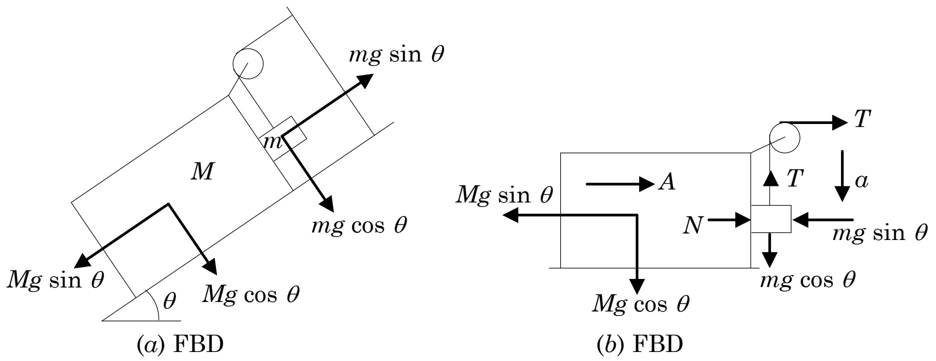

Diagram 5.11 Problem 5.9

Solution: Refer to Diagram 5.11,
Acceleration relation,

$$A = a \qquad [i]$$

Force balance equation of block,

$$mg\cos\theta - T = ma \qquad [ii]$$

$$N - mg\sin\theta \qquad [iii]$$

Force balance equation of wedge,

$$-Mg\sin\theta - N = Ma \qquad [iv]$$

On solving Eqns. (*i*), (*ii*), (*iii*), and (*iv*), we get the acceleration of smaller mass.

Problem 5.10: The three blocks shown in the Diagram 5.12a, move with constant velocity. It is given that velocity of A with respect to C is 300 mm/s upward and that velocity of B with respect to A is 200 mm/s downward. Find out the velocity of each mass with respect to earth.

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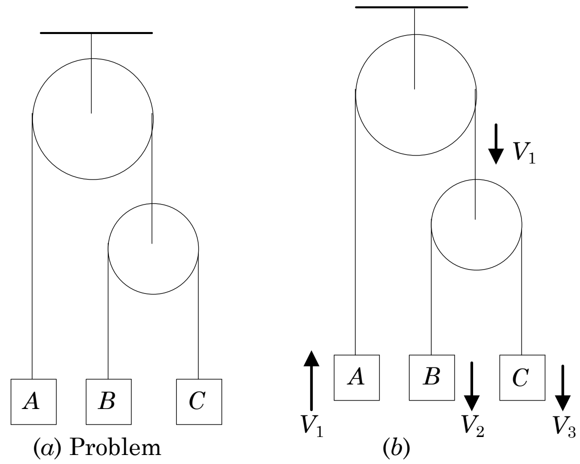

Diagram 5.12 Problem 5.10

Solution: Refer to Diagram 5.12b,

$$V_1 = \frac{V_2 + V_3}{2} \quad [i]$$

$$V_1 + V_3 = 300 \quad [ii]$$

$$V_1 + V_2 = 200 \quad [iii]$$

On solving Eqns. (*i*), (*ii*), and (*iii*), we get the velocity of each mass with respect to earth.

Problem 5.11: The diagram shows three masses, which are initially at the same level. The separation between two consecutive masses is same. When the motion is allowed to start, we find all the three masses to remain on the same straight line. If the acceleration of the central mass is 10 m/s² downwards then find out the acceleration of the other two masses.

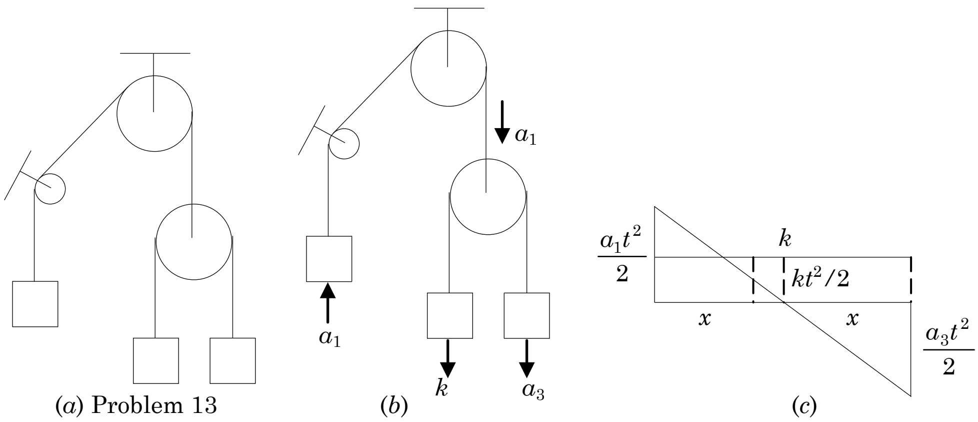

Diagram 5.13 Problem 5.11

Solution: Refer to Diagram 5.13b,

$$a_1 = \frac{a_3 + k}{2} \quad [i]$$

$$\frac{a_1 t^2}{2} + \frac{kt^2}{2} = \frac{a_3 t^2}{2} - \frac{kt^2}{2} \quad [ii]$$

On solving Eqns. (*i*), and (*ii*), with k = 10 m/s², we get the acceleration of the other two masses.

Problem 5.12: In the Diagram 5.14 shown, find out the acceleration of all the masses.

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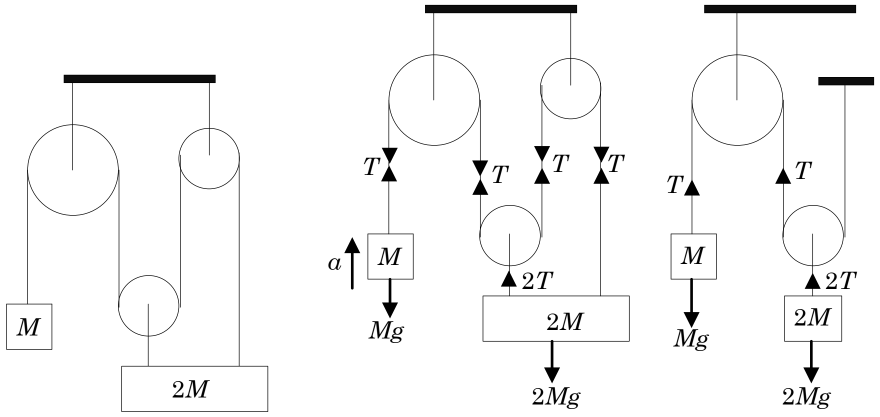


(a) Problem (b) Free body diagram

Diagram 5.14 Problem 5.12

Solution: Refer to Diagram 5.14b, force balance equation,

$$T - Mg = Ma \quad [i]$$

$$2T - 2Mg = 2Ma \quad [ii]$$

On solving Eqns. (i), and (ii), we get the same value of acceleration that is

$$a = \frac{T}{M} - g$$

In the given pulley system all the masses are hanging and the tension to mass ration is same. Than all the masses will say in the state of rest.

Problem 5.13: In the Diagram 5.15 shown, find out the tension in the lower string.

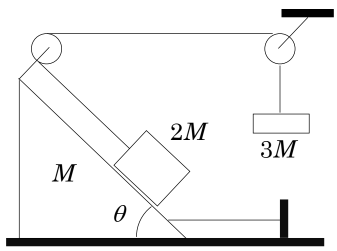
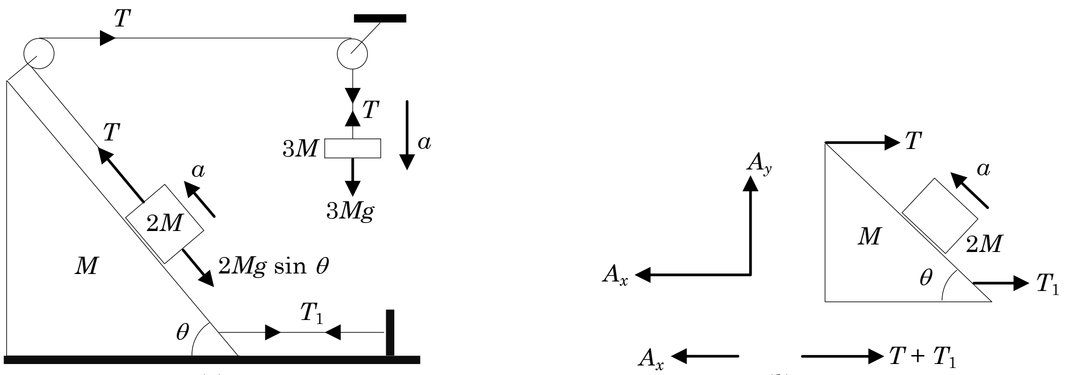


Diagram 5.15 Problem 5.13



(a) (b)

Diagram 5.16 Problem 5.13

Solution: Refer to Diagram 5.15.

Let us assume that the wedge to be in the state of rest. The direction of tension present is string is shown in Diagram 5.16a.

From Diagram 5.16a, force balance equation,

$$T - 2Mg \sin \theta = 2Ma \quad [i]$$

$$3Mg - T = 3Ma \quad [ii]$$

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On solving Eqns. (i), and (ii),

$$a = \frac{Mg\,(3 - 2\sin\theta)}{5M}$$

The acceleration will have +ve value only.

The net horizontal force acting on the system and the acceleration of the centre of mass are in opposite direction. This means that the basic assumption is wrong. The weight will move and T_1 will become equal to zero.

Problem 5.14: The Diagram 5.17 shows a uniform chain of N links. Mass of each link is M. A force $F = NMg + kt$ has been applied as shown, k is a positive constant and t represents time. The joint between the 4th and 5th link can tolerate a tension which is three times the initial value. Find out the time when the breaking will occur.
Draw the variation of tension between the 2nd and the 3rd link as a function of time.

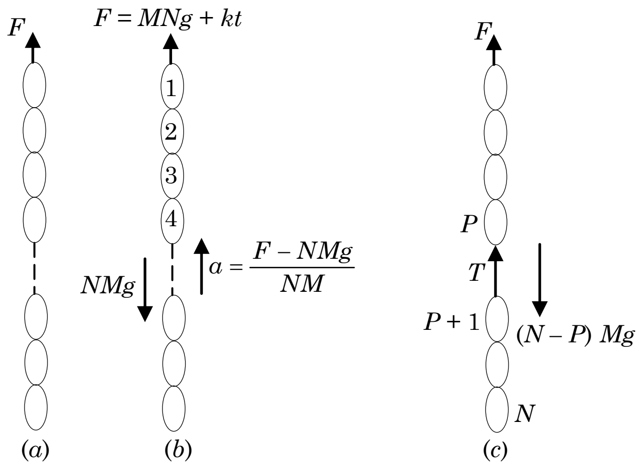

Diagram 5.17 Problem 5.14

Solution: Refer to Diagram 5.17b. Initially the force F is equal to the total weight of the chain.

Acceleration is
$$a = \frac{F - NMg}{NM} = \frac{kt}{Nm}$$

Force at the joint between P and P + 1th link

$$T - (N - P)Mg = (N - P)Ma$$

or,
$$T = (N - P)M\,(a + g)$$

$$T = (N - P)M\left(\frac{kt}{MN} + g\right) \qquad [i]$$

Therefore, initial tension between 4th and 5th link is

$$T = (N - 4)Mg$$

or,
$$3(N - 4)Mg = (N - 4)M\left(\frac{kt_1}{NM} + g\right)$$

or,
$$t_1 = \frac{2gNM}{k}$$

After breaking the chain:

- Acceleration is equal to $-g$,
- Tension is equal to zero,
- First goes up and comes down.

Graph of the tension between 5th and 6th link is as shown in Diagram 5.18a.

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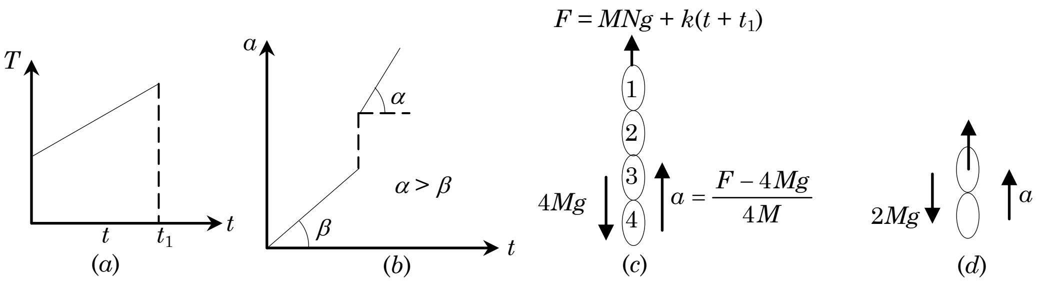

Diagram 5.18 Problem 5.14

Graph of the tension between 2nd and 3rd link is as shown in Diagram 5.18b.
Now, consider variation of time from 0 to t_1,

From Eqn. (*i*), $$T = (N - P)M\left(\frac{kt}{NM} + g\right)$$

Thus, $$T = (N - 2)M\left(\frac{kt}{NM} + g\right)$$

or, $$t_1 = \frac{2gNM}{k}$$

Now, consider variation of time after t_1, Refer to Diagram 5.18c and d.

$$F = (N - 2)Mk(t + t_1)$$

Now, acceleration is $$a = \frac{F - 4Mg}{4M}$$

Thus,

$$a = \frac{3NMg + kt - 4Mg}{4M} \quad [ii]$$

Consider force balance equation [Diagram 5.18d],

$$T = 2Mg + 2Ma \quad [iii]$$

On solving Eqns. (*ii*), and (*iii*),

$$T = 2M\left[\frac{3NMg + kt - 4Mg}{4M}\right] + 2Mg \quad [iv]$$

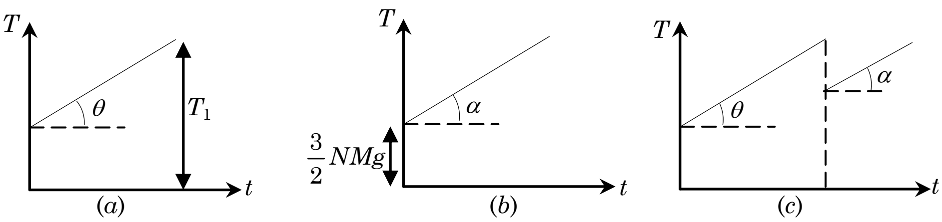

Diagram 5.19 Problem 5.14

Variation before breaking the chain is as shown in Diagram 5.19a.

Thus, $$T_1 = (N - 2)M \times 3g$$

and, $$\tan\theta = \frac{(N - 2)k}{N}$$

Variation after breaking the chain is as shown in Diagram 5.19b. Tension between the 2nd and the 3rd link immediately after the chain breaks,

From Eqn. (*iv*), $$T = \frac{3NMg + k \times 0 + 4Mg}{2} + 2Mg$$

or, $$T = \frac{3}{2}NMg$$

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thus, $\tan \alpha = k/2$.

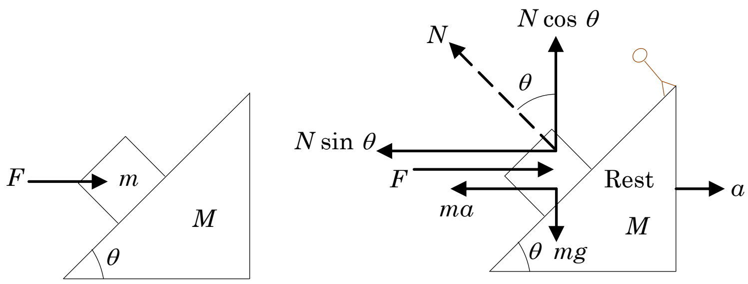
In between 2nd and 3rd link tension more before chain breaks, and tension is less after chain breaks.

If $N \geq 5$, thus $\frac{(N-2)k}{N}$, or, $\left(1-\frac{2}{N}\right)k$

Therefore, $\tan\theta = \frac{3}{5}k$

This is the minimum value of tan θ.

Problem 5.15: In the Diagram 5.20 shown, find out the value force F such that no relative motion takes place between the wedge and the block.



(*a*) Problem 25 (*b*) FBD of block

Diagram 5.20 Problem 5.15

Solution: Refer to Diagram 5.20b,

Acceleration, $$a = \frac{F}{M+m} = \frac{2(M+m)g\tan\theta}{M+m} = 2g\tan\theta$$

Consider free body diagram of the block with respect to wedge,

$$N\cos\theta = mg \qquad [i]$$

$$F = ma + N\sin\theta \qquad [ii]$$

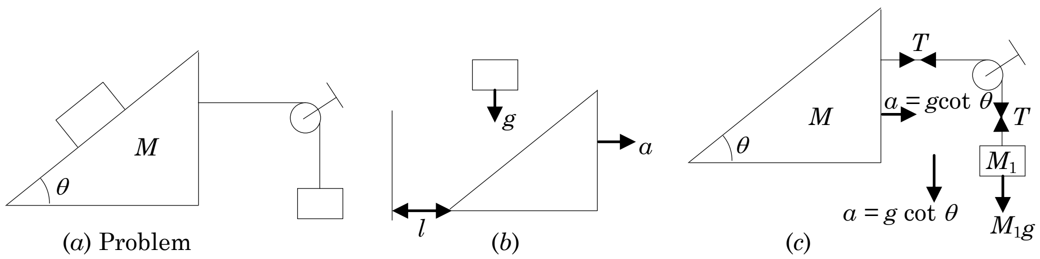
On solving Eqns. (*i*), and (*ii*),

$$F = ma + N\sin\theta$$

or, $$F = m\frac{F}{M+m} + mg\tan\theta$$

or, $$F = \left(\frac{M+m}{M}\right)mg\tan\theta$$

Problem 5.16: In the Diagram 5.21 shown, find out the minimum value of the mass of the hanging block such that the block present on the wedge looses contact. Mass of the wedge is M. Mass of the block present on the wedge is M.



(*a*) Problem (*b*) (*c*)

Diagram 5.21 Problem 5.16

Solution: Here we find out acceleration with which the wedge should be move, so that the normal reaction between the block and the wedge becomes zero.

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If the block has zero normal reaction than it will really fall under gravity in th time in which the block falls down a distance equal to height of the wedge. The wedge must move horizontally distance is equal to length of its base.

Refer to Diagram 5.21b,

$$l = \frac{at^2}{2}, \text{ and } h = \frac{gt^2}{2}$$

Thus, $$\tan\theta = \frac{h}{l} = \frac{g}{a}$$

Refer to Diagram 5.21c,

$$M_1 g - T = M_1 a \quad [i]$$

$$M_1 g = (M + M_1)\, a \quad [ii]$$

or, $$M_1 g - M_1 a = Ma$$

or, $$M_1 = \frac{Ma}{g-a} = \frac{M}{\frac{g}{a} - 1} = \frac{M}{\tan\theta - 1}$$

The problem is correct only when θ is less than 45°.

Problem 5.17: In the Diagram 5.22 shown, the sphere remains in contact with both the wedges. It is given that the sphere is coming down with acceleration A. Find out the acceleration of both the wedge and the total acceleration of the sphere.

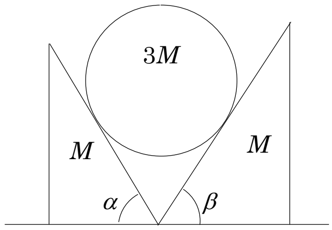

Diagram 5.22 Problem 5.17

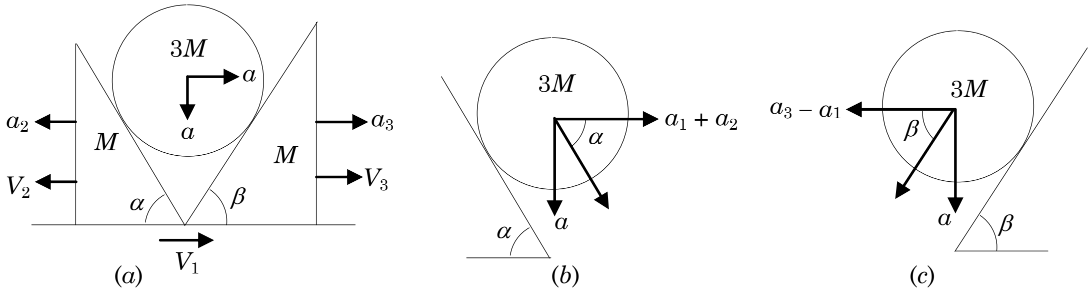

Diagram 5.23 Problem 5.17

Solution: Refer to Diagram 5.23a,

Consider the two wedges and the sphere inside a system boundary, no any external force acts on the system. This means that linear momentum must remains conserved in the horizontal direction.

$$3MV_1 + MV_3 - MV_2 = 0$$

or, $$3V_1 + V_3 - V_2 = 0$$

differentiating with respect to time,

$$3a_1 + a_3 - a_2 = 0 \quad [i]$$

Refer to Diagram 5.23b,

$$\tan\alpha = \frac{a}{a_1 + a_2} \quad [ii]$$

Refer to Diagram 5.23c,

$$\tan\beta = \frac{a}{a_3 - a_1} \quad [iii]$$

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On solving Eqns. (*i*), (*ii*), and (*iii*), acceleration of both the wedge and the total acceleration of the sphere.

Problem 5.18: Three identical spheres have been placed on a horizontal surface. A belt has tied the spheres. The belt has zero tension initially. Now, a fourth identical sphere is place symmetrically on the top of the three spheres. Find out the tension in the belt.

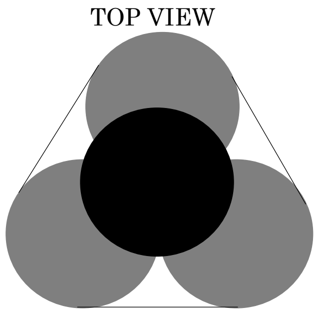

Diagram 5.24 Problem 5.18

Solution: Horizontal plane and lower surface: Refer to Diagram a, the center of the fourth sphere is vertically above the point O.

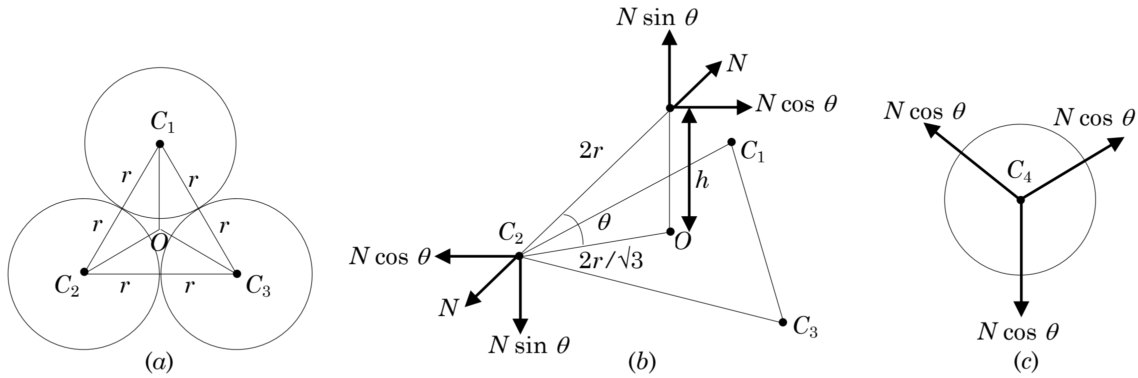

Diagram 5.25 Problem 5.18

Thus, from Diagram 5.25a, $$C_2O = \frac{2r}{\sqrt{3}}$$

Consider the free body diagram 5.25b. the total vertical force acting on the upper sphere is $3N \sin \theta$, and it will balance the weight of thus upper sphere.

$$3N \sin \theta = Mg \qquad [i]$$

Refer to Diagram 5.25c, represents the top view of the upper sphere. Here we see that the components of $N \cos \theta$, will be cancelled. Only $N \sin \theta$, acting on the lower surface (C_2) will be balance by the normal reaction from the ground.

Let us suppose the top view of a lower surface [Diagram 5.25d]. force balance equation

$$2T \sin 60° = N \cos \theta \qquad [ii]$$

On solving Eqns. (*i*), and (*ii*),

$$2T \sin 60° = \frac{Mg}{3 \sin \theta} \cos \theta$$

or, $$T = \frac{Mg}{3 \times 2 \sin 60°} \cot \theta$$

From diagram $\cot \theta = 1/\sqrt{2}$, thus

Thus, $$T = \frac{Mg}{3\sqrt{6}}$$

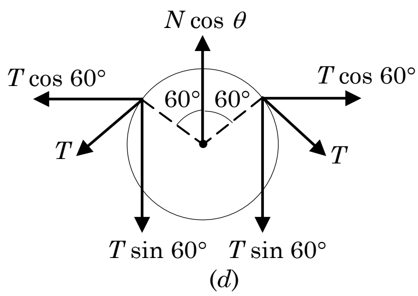

Diagram 5.25 Problem 5.18

- Tension on the belt does not depend on the masses of lower sphere.
- There cannot be any normal reaction between the lower three sphere because the normal reaction between them was zero initially.

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