Solved Problems of Mechanics
Chapter–4 Projectile Motion

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Problem 4.1: If for a projectile u is the speed of projection, R is the horizontal range, t is the time of flight and h is the maximum height, prove the following relations

$$u^2 = 2g[H + R^2/16h]$$

Solution: The projectile thrown from the ground and it's finally fall on the ground.

$$u^2 = 2g[h + R^2/16h]$$

or,
$$= 2g\left[\frac{u^2\sin^2\theta}{2g} + \frac{\frac{u^2\sin 2\theta}{g}}{16\frac{u^2\sin^2\theta}{2g}}\right]$$

or,
$$= 2g\frac{u^2\sin^2\theta + u^2\cos^2\theta}{2g}$$

or,
$$= u^2$$

Problem 4.2: A particle is projected with initial velocity u at an angle θ with the horizontal. After how much time will the direction of velocity vector become perpendicular to the initial velocity vector?

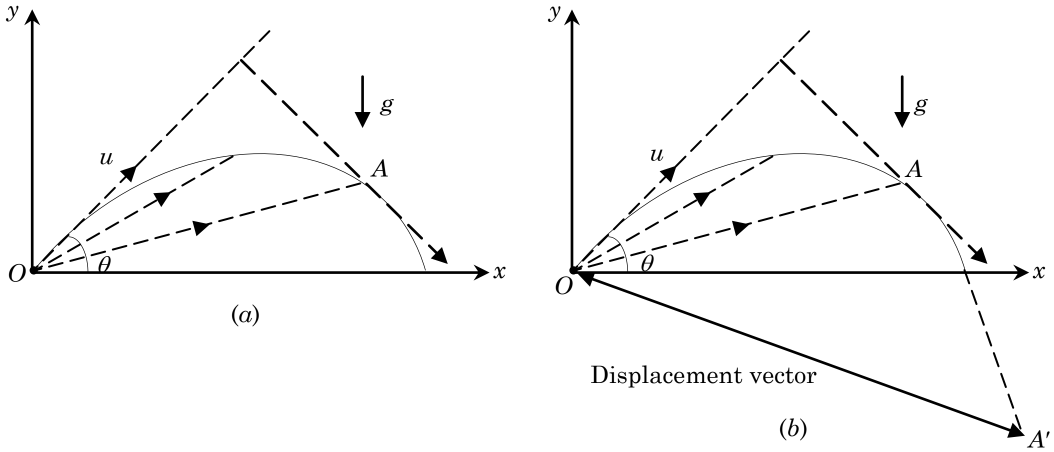

Diagram 4.1 Problem 4.2

Solution: Refer to Diagram 4.1a.

Initial velocity, $V_i = u\cos\theta\, i + u\sin\theta\, j$,

Final velocity, $V_f = u\cos\theta\, i + (u\sin\theta - gt)\, j$,

Since, $V_i\, V_f = 0$.

Thus, $u^2\cos^2\theta + u^2\sin^2\theta - ugt\sin\theta = 0$

or, $u^2 - ugt\sin\theta = 0$

or, $t = u/(g\sin\theta)$

Average velocity: Refer to Diagram 4.1b. The displacement vector from O to A_1 is perpendicular to the initial velocity. Therefore V_{avg} from O to A_1 is perpendicular to the initial velocity.

From Diagram 4.1b, $x = tu\cos\theta,\ y = tu\sin\theta - \frac{1}{2}gt^2$,

Thus, $R = x\, i + y\, j$,

So, $V_i\, R = 0$

or, $u\cos\theta\, x + u\sin\theta\, y = 0$

or, $$u\cos\theta \cdot u\cos\theta\, t + u\sin\theta\left(u\sin\theta\, t - \frac{1}{2}gt^2\right) = 0$$

or, $$t = \frac{2u}{g\sin\theta}$$

The time in which velocity vector becomes perpendicular to the initial velocity vector is 1/2 of the time in which V_{avg} become perpendicular to the initial velocity vector.

Problem 4.3: Two projectiles are projected from the same point at the same time with initial velocity u and V and at angle α and β respectively. Prove that after time $t = uv\sin(\alpha - \beta) / [g(V\cos\beta - u\cos\alpha)]$ their velocity vectors become parallel.

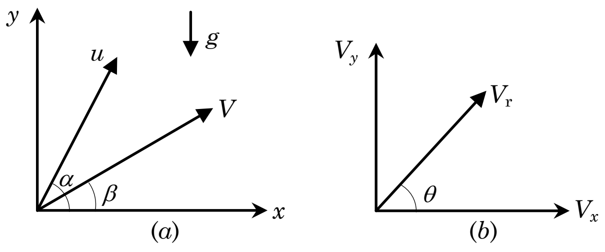

Diagram 4.2 Problem 4.3

Solution: If two projectiles thrown from the same position and different angle then the relative separation will continuously increase with respect to time. Refer to Diagram 4.2a. With respect to particle 1 the particle 2 will move on a straight line and it will always move away from the particle 1.
Refer to Diagram 4.2b,

$$V_1 = u\cos\alpha\, i + (u\sin\alpha - gt)\, j,$$

and,

$$V_2 = V\cos\beta\, i + (V\sin\beta - gt)\, j,$$

Thus,

$$\frac{u\sin\alpha - gt}{u\cos\alpha} = \frac{V\sin\beta - gt}{V\cos\beta}$$

or,

$$|V_1 \times V_2| = 0$$

If the velocity vector are parallel then it does not implies that two projectiles are moving in the same direction.

Problem 4.4: A particle is projected from the origin with its initial velocity components along horizontal and vertical to be as u_1 and u_2 respectively. The particle passes through the point (h, k). Prove that $2u_1^2\,2k + gh^2 = 2u_1u_2h$.

Solution: Since,

$$u_1 = u\cos\theta, \text{ and, } u_2 = u\sin\theta,$$

Thus,

$$y = \frac{u_2}{u_1}x - \frac{1}{2u_1^2}gx^2$$

Since, the particle passes through the point (h, k),

so,

$$k = \frac{u_2}{u_1}h - \frac{1}{2u_1^2}gh^2$$

or,

$$2u_1^2\,2k + gh^2 = 2u_1u_2h$$

Problem 4.5: A projectile is to hit a target having coordinates (h, k). The projectile is fired from the origin with a velocity $[2gh]^{1/2}$. Show that it is impossible to hit the target if $k > 3h/4$.

Solution: Since, path is

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

or,

$$\frac{gx^2}{2u^2}\tan^2\theta - x\tan\theta + y - \frac{gx^2}{2u^2} = 0$$

or,

$$\frac{gh^2}{2u^2}\tan^2\theta - h\tan\theta + k - \frac{gh^2}{2u^2} = 0$$

The condition for the projectile to not pass (h, k) is that D of the above quadratic equation is less than zero.

Thus,

$$h^2 - \frac{4gh^2}{2u}\left(k + \frac{gh^2}{2u^2}\right) < 0$$

or,

$$k > 3h/4$$

Problem 4.6: The Diagram 4.3a shows trajectories of three projectiles *A*, *B* and *C*. Arrange in the increasing order their

a) time of flight
b) initial vertical component of velocity.
c) projection speed.

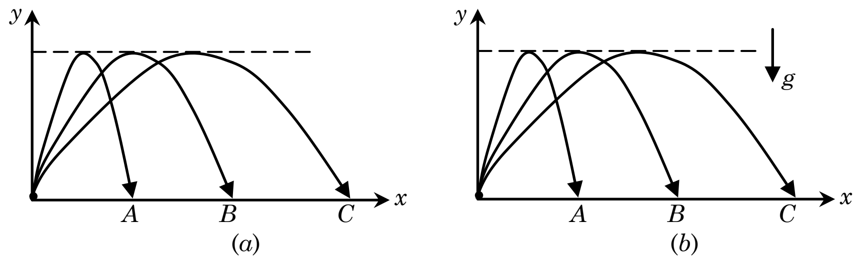

Diagram 4.3 Problem 4.6

Solution: Since the three projectiles at the same in a height therefore the initial vertical component of velocity is same for *A*, *B*, *C*.

The formula of time of flight is $\frac{2u\sin\theta}{g}$. Therefore $T_A = T_B = T_C$.

Range of *C* is maximum and range of *A* is minimum.
Horizontal velocity or initial horizontal velocity is maximum of *C*, and minimum of *A*.

$$u = \sqrt{u_x^2 + u_y^2}$$

$$= u_C > u_B > u_A.$$

Initial vertical velocity of *A* is maximum and *C* is minimum, $T_A > T_B > T_C$.
Time of *C* is minimum and horizontal range is maximum. Therefore initial horizontal velocity of *C* is maximum and of *A* is minimum.

Thus, $R = u_x t$.

or, $u_x = R/t$.

Compression of *D* and *C*: $T_D > T_C$, $R_C = T_D$.

or, $\frac{R_D}{T_D} < \frac{R_C}{T_C}$

or, $u_{xD} < u_{xC}$.

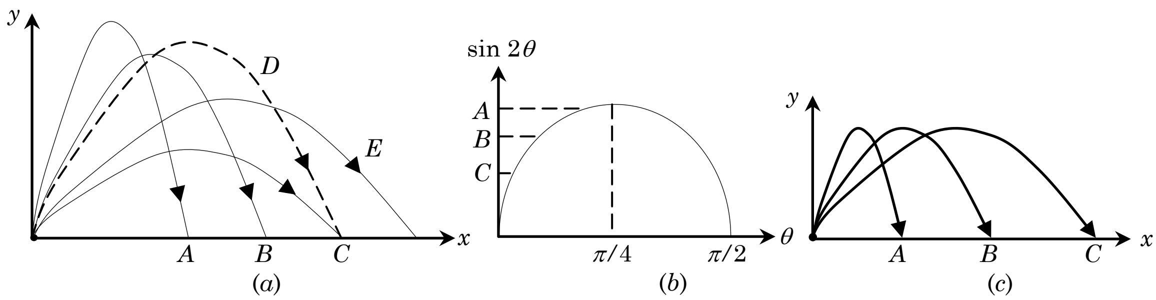

Diagram 4.4 Problem 4.6

Compression of *E* and *C*: We cannot compare. In compression *C* with *E* takes more time, then range and horizontal distance is more as compare to *C*.
Thus, we cannot compare the initial projectile speed of *A*, *B* and *C*.

For *A*, $u = \sqrt{u_x^2 \downarrow + u_y^2 \uparrow}$

For *C*, $u = \sqrt{u_x^2 \uparrow + u_y^2 \downarrow}$

If the angle of projection is less than 45° for AB then we can not compare to the initial projection speed.

$$u^2 = \frac{gR}{\sin 2\theta}$$

If θ is lies between 0 to $\pi/4$, then sin 2θ will be an increasing function by the help of formula we can say that u_C is maximum and u_A is minimum.

or, $$u_{xC} > u_{xB}.$$

We cannot compare u_{xC} and u_{xB}, and we cannot compare u_{xB} and u_{xA}.

Problem 4.7: A particle is projected from the origin O at an angle α; it passes through a point P and strikes the horizontal plane at point Q. If $\angle POQ = \gamma$ and $\angle PQO = \beta$ then prove that

$$\tan \alpha = \tan \beta + \tan \gamma.$$

Solution: Refer to Diagram 4.5.

Since, $$\tan \alpha = \tan \beta + \tan \gamma$$

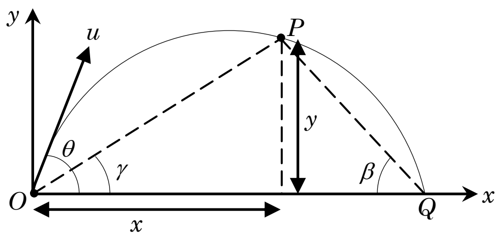

Diagram 4.5 Problem 4.7

or, $$\tan \alpha = \frac{y}{R-x} + \frac{y}{x}$$

or, $$\tan \alpha = \frac{yR}{x(R-x)}$$

or, $$y = x \tan \alpha - \frac{x^2 \tan \alpha}{R}$$

or, $$y = x \tan \alpha - \frac{x^2 \tan \alpha}{u^2 \sin 2\alpha / g}$$

or, $$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

We cannot assume the result to be prove that first equation. This equation will holds true even of the point P is lies blow the x-axis.

Problem 4.8: A regular hexagon stands up with one side on the ground and a particle is projected so as to graze its four vertices. Show that the velocity of the particle on reaching the ground is to the least velocity as √31 to √3.

Solution: The ratio of minimum velocity to maximum velocity is cos θ. Considering the starting point to be the origin the equation of the parabola will be

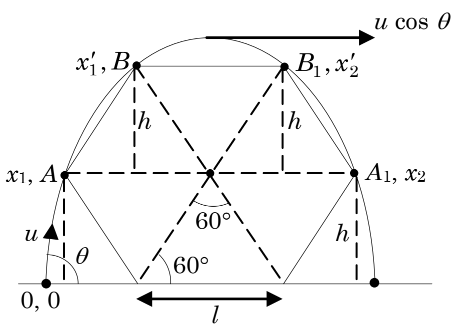

Diagram 4.5 Problem 4.8

$$y = ax - bx^2,$$

or, $$h = ax - bx^2,$$

or, $$bx^2 - ax + h,$$

Thus, difference in roots are $x_2 - x_1 = 2l$.

and, $$2h = ax - bx^2,$$

or, $$bx^2 - ax + 2h,$$

Thus, difference in roots are $x'_2 - x'_1 = l$.

Therefore, $$(x_2 - x_1)^2 = (x_2 + x_1)^2 - 4x_1x_2$$

$$2l^2 = \left(\frac{a}{b}\right)^2 - 4\frac{h}{b} \qquad [i]$$

$$l^2 = \left(\frac{a}{b}\right)^2 - 8\frac{h}{b} \qquad [ii]$$

From Eqns. (*i*), and (*ii*), $b = 4h/3l^2$.
From Eqn. (*i*) × 2, and subtracting,

$$2\times 2l^2 - l^2 = 2\left(\frac{a}{b}\right)^2 - \left(\frac{a}{b}\right)^2$$

or, $$a^2 = b^2 \times 7l^2$$

or, $$a = \frac{4h\sqrt{7}}{3l}$$

Since, $$h = \sqrt{3}l/2$$

Thus, $$a = \sqrt{\frac{28}{3}}\tan\theta$$

Now, $$\frac{V_{max}}{V_{min}} = \frac{1}{\cos\theta} = \sqrt{1+\tan^2\theta}$$

or, $$= \frac{\sqrt{31}}{\sqrt{3}}$$

Problem 4.9: A gun is placed on the top of a tower of height *h*. The tower is surrounded by sea. In the sea, a ship is present having a gun similar to the gun placed on the top of the tower. Both guns have the same projection speed equal to $[2gk]^{1/2}$.
Show that there is a region of area $8\pi hk$ in the sea in which the gun placed in the tower is out of the range of the gun present in the ship but the gun present in the ship is in the range of the gun present in the tower.

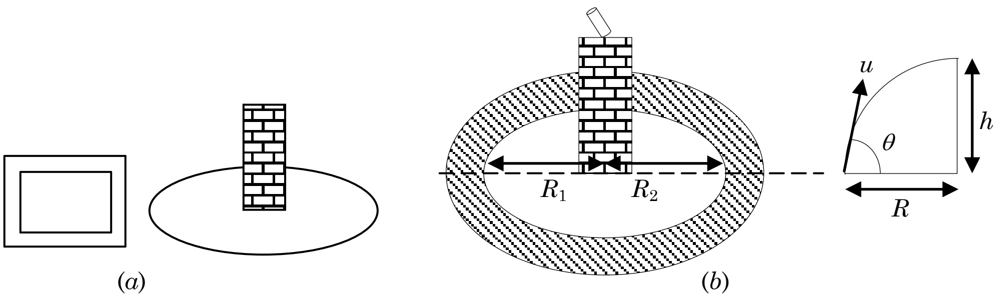

Diagram 4.6 Problem 4.9

Solution: The shaded portion [Diagram 4.9b] show the region of gun when gun of the fire cannot hit the gun placed in the tower. But it cannot hit by the gun placed in the tower.

$$R_2 = \sqrt{\frac{2u^2}{g}\left(\frac{u^2}{2g}+h\right)}, \text{ and } R_1 = \sqrt{\frac{2u^2}{g}\left(\frac{u^2}{2g}-h\right)}$$

Thus, $$R = \pi(R_2^2 - R_1^2) = 4\pi\frac{u^2h}{g} = 8\pi h$$