Solved Problems of Mechanics
Chapter 2 - Motion on Straight Line

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Problem 2.1: A car accelerates from rest with at a rate *A* for sometime. After which it decelerates at a rate '*B*' for sometime till it comes to a state of rest. If the total time of motion is *T* then find out the distance travelled by the car.

Solution: Refer to Diagram 2.1.

Motion of the body from *P* to *Q*:

$$V - 0 = AT_1, \qquad [i]$$

$$S_1 = PQ = 0 \times T_1 + AT_1^2/2 \qquad [ii]$$

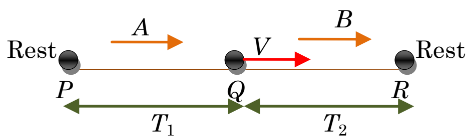


Diagram 2.1 Problem 2.1

Motion of the body from *Q* to *R*:

$$0 - V = BT_2, \qquad [iii]$$

$$S_2 = QR = VT_2 - BT_2^2/2$$

$$S_2 = BT_2 - BT_2^2/2 = BT_2^2/2 \qquad [iv]$$

Now, total time is $T = T_1 + T_2$,

Now, we see that $AT_1 = BT_2$, then from above equation

$$BT_2 + AT_2 = AT$$

After solving, we get $T_2 = \dfrac{AT}{A+B}$, $T_1 = \dfrac{BT}{A+B}$

Using Eqns. (*ii*), and (*iv*), we get

$$S_1 = \frac{AB^2T^2}{2(A+B)^2}, \quad S_2 = \frac{BA^2T^2}{2(A+B)^2}$$

Total travelled distance is $S = S_1 + S_2$,

$$S = S_1 + S_2 = \frac{AB^2T^2}{2(A+B)^2} + \frac{BA^2T^2}{2(A+B)^2} = \frac{ABT^2}{2(A+B)}$$

Problem 2.2: A particle P starts moving on a straight line with speed u and acceleration a. One second later, another particle Q starts from the same point with speed $u/2$ and acceleration $2a$ on the same straight line. When Q crosses P, their velocities are 31 m/s and 22 m/s respectively. Find out the distance travelled by P.

Solution: The motion of Q has two disadvantages:

1. It start moving 1 sec late,
2. Its initial velocity is 1/2 of the velocity of particle P.

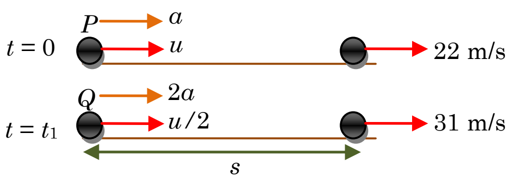


Diagram 2.2 Problem 2.2

Let time taken by P to cover a distance S be equal to T, then time taken by Q will be $(T - 1)$ sec.

For particle P,

$$22^2 - u^2 = 2aS \qquad [i]$$

For particle Q,

$$31^2 - (u/2)^2 = 2 \times 2aS \qquad [ii]$$

From Eqns. (i), and (ii),

$$31^2 - (u/2)^2 = 2 \times (22^2 - u^2)$$

After solving, we get $u = 2$ m/sec.

Again, for particle P,

$$22 - u = aT. \qquad [iii]$$

For particle Q,

$$31 - (u/2) = 2a(T - 1). \qquad [iv]$$

After solving, Eqns. (iii), and (iv), we get

$$T = 4 \text{ sec, and } a = 5 \text{ m/sec.}$$

Problem 2.3: A man with constant reaction time can stop his car within 30 m when it is moving at a speed of 72 km/hr and within 10 m when it is moving at a speed of 36 km/hr. What is the distance within which he can stop the car if it is travelling at a speed of 54 km/hr?

Solution: In this problem the reaction time on driver and the retardation on applied the break as the same value for all cases.

Now,

$$30 \text{ m} = 20\ T_1 \times + 20^2/2a, \qquad [i]$$

$$10 \text{ m} = 10\ T_1 \times + 10^2/2a, \qquad [ii]$$

After solving Eqns. (i), and (ii), we get

$$T_1 = 0.5 \text{ sec.,} \quad \text{and} \quad a = 10 \text{ m/sec}^2.$$

Save distance is

$$= 15\ T_1 \times + 15^2/2a,$$

$$= 15 \times 0.5 + 15^2/(2 \times 10) = 18.75 \text{ meter.}$$

Problem 2.4: It takes 2 minutes to acquire full speed of 60 km/hr from rest and exactly 1 minute to come to rest from full speed. If somewhere in between two stations 1 km of the track is being repaired and the speed limit is 20 km/hr, how late will the train be due to repair works?

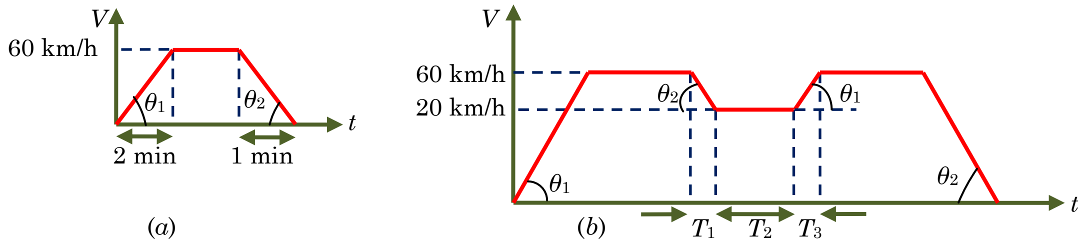


Diagram 2.3 Problem 2.4

Solution: The train will start from rest and finally it will come to rest. The acceleration and the retardation of train will have same value for any station.

From Diagram 2.3a, $\tan\theta_1 = a$.

$$\tan\theta_1 = a = \frac{60\times 5/18}{120} = \frac{5}{36}\ \text{m/s}^2.$$

and, $\tan\theta_2 = a_1$.

$$\tan\theta_2 = a_1 = \frac{60\times 5/18}{60} = \frac{5}{18}\ \text{m/s}^2.$$

Now, T_1 is the time which the train takes to cover that part of track which is repair.

$$T_1\times 20\ \text{km/h} = 1\ \text{km}.$$

$$T_1 = 1/20\ \text{hour} = 180\ \text{sec}.$$

Now, T_2 is the time which the train takes to reduce its speed from 60 km/h to 20 km/h.

From Diagram 2.3b, $\tan\theta_2 = 40/T_2$.

$$T_2 = \frac{40\times(5/18)}{5/18} = 40\ \text{sec}.$$

Again, $$\tan\theta_1 = \frac{5}{36} = \frac{40\times 5/18}{T_3}\ \text{m/s}^2.$$

$T_3 = 80$ sec.

The total shaded area shown in the graph will give the distance which the train covered at a speed of 60 km/h.

Now, area 1 is

$$s_2 = \frac{5}{18}\left(\frac{60+20}{2}\right)T_2 = \frac{5}{18}\left(\frac{60+20}{2}\right)\times 40 = \frac{5\times 1600}{18}\ \text{m}.$$

$s_1 = 1000$

$$s_3 = \frac{5}{18}\left(\frac{60+20}{2}\right)T_3 = \frac{5}{18}\left(\frac{60+20}{2}\right)\times 80 = \frac{5\times 3200}{18}\ \text{m}.$$

Total distance is $s = s_1 + s_2 + s_3 = 7000/3$ meter.

This distance was covered by the train at a speed of 60 km/h.

Now, total time is $$T = \frac{7000}{60\times 5/18} = 140\ \text{sec}.$$

$$= T_1 + T_2 + T_3 = 300\ \text{sec}.$$

Now, extreme time is

$$= 300 - 140 = 160\ \text{sec.} = 2\ \text{min}\ 40\ \text{sec}.$$

Now assumptions in the problems are:

The one km length of the track lies between the two stations such that the train can get the maximum speed before reaching the track and also after leaving the track.

Problem 2.5: A particle is thrown up with an initial velocity u. If after T_1 and T_2 time the particle is at height H, then prove that

$$H = gT_1T_2/2;\ u = g(T_1 + T_2)/2;\ \text{Maximum height} = g(T_1 + T_2)^2/8$$

Solution: Since, we know that

$$H = ut - \frac{1}{2}gt^2$$

or,
$$gt^2 - 2ut - 2H = 0$$

Product of roots of the above quadratic equation is

$$T_1T_2 = 2H/g,$$

Then, we get $H = gT_1T_2/2$.

Now, sum of roots of the above quadratic equation is

$$T_1 + T_2 = 2u/g,$$

Then, we get $u = g(T_1 + T_2)/2$.

Now, maximum height = $u^2/2g$.

or,
$$= \frac{g\left(\frac{T_1 + T_2}{2}\right)^2}{2g}$$

Maximum height = $g(T_1 + T_2)^2/8$.

Problem 2.6: A body is projected vertically upwards from point A, top of a tower. It reaches the ground in T_1 seconds. If it is projected vertically downwards from A with the same initial velocity, it takes T_2 seconds. Show that if we allow it to fall freely under gravity then it should take $[T_1T_2]^{1/2}$.

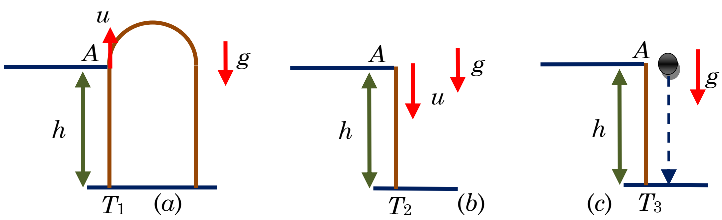


Diagram 2.4 Problem 2.6

Solution: From Diagram 2.4(a),

$$-h = uT_1 - \frac{1}{2}gT_1^2 \qquad [i]$$

From Diagram 2.4(b),

$$-h = -uT_2 - \frac{1}{2}gT_2^2 \qquad [ii]$$

From Diagram 2.4(c),

$$-h = 0 \times T_3 - \frac{1}{2}gT_3^2 \qquad [iii]$$

Multiplying in Eqn. (i) by T_1, and in Eqn. (ii) by T_2, and then adding

$$-h(T_1 + T_2) = -gT_1T_2\left(\frac{T_1 + T_2}{2}\right)$$

or, $h = g\, T_1T_2/2$

Put this value in Eqn. (*iii*), then we get

$$-\frac{1}{2}gT_1T_2 = -\frac{1}{2}gT_3^2$$

Then, we get $[T_1T_2]^{1/2}$.

Problem 2.7: A particle is projected vertically upwards, and T seconds afterwards another particle is projected upwards with the same initial velocity u. Prove that the two particles will meet after time $T/2 + u/g$. Also find the velocity of the two particles when they meet.

Solution: Both the particle has been projected with the same initial velocity when the two particles will be u, then they will be moving in opposite direction.

The extra time of particle 1 is compression to particle 2 is used by 1 to go from P to P.

Total time taken by particle 1 to meet particle 2 is

$$\frac{u}{g} + \frac{T}{2}$$

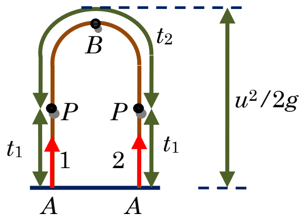


Diagram 2.5 Problem 2.7

Total time taken by particle 2 to meet particle 1 is

$$t_1 = \frac{u}{g} - \frac{T}{2}$$

Velocities are $V = u - gt$.

$$V = u - g\left(\frac{u}{g} + \frac{T}{2}\right) = -\frac{1}{2}gT$$

and, $$V = u - g\left(\frac{u}{g} - \frac{T}{2}\right) = +\frac{1}{2}gT$$

Problem 2.8: A particle is dropped from the top of a tower. After the particle has fallen a distance x, another particle is dropped from a distance y below the top. If both the particles reach the ground in the same time, prove that the height of the tower is equal to $(x + y)^2/4x$.

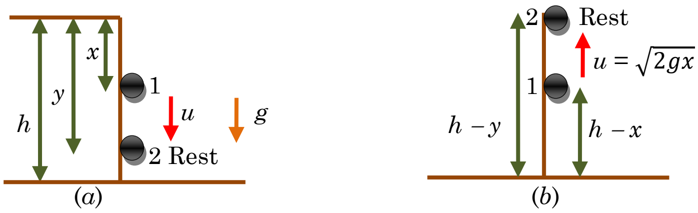


Diagram 2.6 Problem 2.8

Solution: Both the particles reach the ground in the same time. When the particle 2 has been drop the particle 1 moving under some velocity. If they have to reach the ground together then y should be greater than x.

The vector method is useful only in the question in which a body changes its direction of motion. If the motion is taking place only in one direction then we can solve by ordinary motion.

$$u^2 - 0^2 = 2gx,$$
$$u = [2gx]^{1/2},$$

For particle 1

$$h - x = ut - \frac{1}{2}gt^2 \qquad [i]$$

For particle 2

$$h - y = \frac{1}{2}gt^2 \qquad [ii]$$

Then we get $h - y = gt^2/2.$

$y - x = ut.$

$t = (y - x)/u.$

Using Eqn. (*ii*), then

$$h - y = \frac{1}{2}g\left(\frac{y-x}{u}\right)^2 = \frac{1}{2}g\frac{(y-x)^2}{2gx}$$

or, $$h = \frac{(y+x)^2}{4x}$$

In the Diagram 2.6b we have change the states of two particles. They will again reach the ground in the same time.

Problem 2.9: A particle moves under a constant acceleration and it covers distance *a, b* end *c* the p^{th}, q^{th} and r^{th} respectively.

Prove that $a(q - r) + b(r - p) + c(p - q) = 0.$

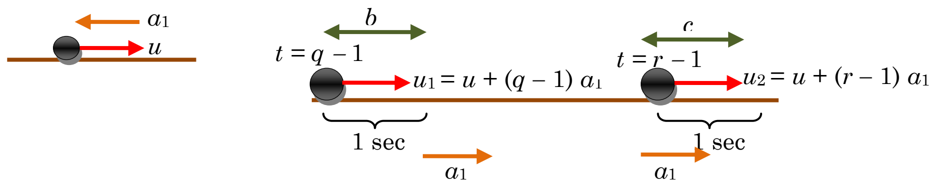


Diagram 2.7 Problem 2.9

Solution: Using Eqn. (2.17)

$$a = u + \frac{1}{2}a_1(2p-1), \quad b = u + \frac{1}{2}a_1(2q-1), \quad c = u + \frac{1}{2}a_1(2r-1)$$

Now,

$$b - c = a_1(q - r), \quad c - a = a_1(r - p), \quad a - b = a_1(p - q)$$

Using Diagram 2.7b,

$$b = u_1 \times 1 + \frac{1}{2}a_1 \times 1^2, \quad c = u_1 \times 1 + \frac{1}{2}a_1 \times 1^2$$

$$c - b = (u_2 - u_1) \times 1; \quad \text{and} \quad c - b = a_1(r - q),$$

If n^{th} *m* sec

$$c - b = (u_2 - u_1) \times m;$$

$$u_1 = u + m(q - 1)a_1, \quad u_2 = u + m(r - 1)a_1,$$

$$u_2 - u_1 = m(r - q)a_1,$$

then, $$c - b = m^2(r - q)a_1;$$

The above formula or result will be true even, if we say that *a, b*, and, *c* are the distance travel in the p^{th}, q^{th}, and r^{th} *m* sec.

Problem 2.10: A particle is thrown vertically upwards and the resistance due to air is not neglected. Assume that air provides constant retardation *K* m/s² always acting opposite to the direction of motion. Show that the time taken to rise is less than the time taken to fall.

If the initial velocity is V m/s than find out the velocity with which the particle returns back to the starting point.

Solution: Now, first of all we taken that the air resistance is constant, the resistance to a motion can give in three different forms:

1. Resistance due to air,
2. Friction due to air,
3. Viscous force due to liquid.

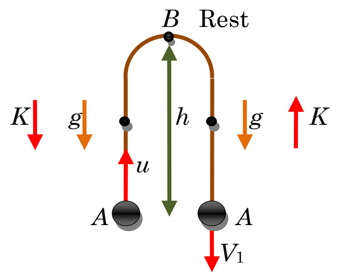


Diagram 2.8 Problem 2.10

These resistance force acts due to relative motion only. When the body goes from A to B, then the retardation of body $g + K$, and when it comes from B to A, then the acceleration will be $g - K$.

Time taken to go from A to B,

$$0 - u = -(g + K)\, t_1,$$

or, $$t_1 = u/(g + K),$$

or, $$t_1 = \frac{u}{\sqrt{(g+K)^2}}$$

$$0^2 - u^2 = 2 \times (g + K)h,$$

or, $$h = \frac{u^2}{2\,(g+K)}$$

Motion from B to A,

$$h = ut_2 + \frac{1}{2}(g-K)\,t_2^2$$

or, $$t_2 = \sqrt{\frac{2h}{g-K}} = \frac{u}{\sqrt{(g-K)\,(g+K)}}$$

By compression we can say that t_2 is greater than t_1.

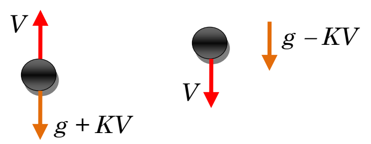
Velocity of the body when it come back to ground.

$$V_1^2 - 0 = 2(g-K)h$$

or, $$V_1 = u\sqrt{\frac{g-K}{g+K}}$$

The final kinetic energy is less than the initial kinetic energy because heat is generated during the motion.

The average heat generation is



$$\frac{\frac{1}{2}mu^2 - \frac{1}{2}mV_1^2}{t_1 + t_2}$$

Now, air resistance is directly proportional to relative velocity, then

$$-\frac{dV}{dt} = g + KV$$

On integration $$\int_u^V \frac{dV}{g+KV} = \int_0^t dt$$

or, $$-\frac{1}{K}\ln\frac{g+KV}{g+Ku} = t$$

or,
$$V = \frac{(g + KV)\, e^{-Kt} - g}{K}$$

Time in which the velocity of particle becomes equal to zero, when it is going upward
$$-\frac{1}{K}\ln\frac{g + Ku}{g} = t_1$$

Velocity of the particle when it come back. The particle will have instantaneous acceleration of $g - KV$, then
$$\frac{dV}{dt} = g - KV$$

On integration
$$\int_0^V \frac{dV}{g - KV} = \int_0^t dt$$

or,
$$-\frac{1}{K}\ln\frac{g - KV}{g} = t$$

or,
$$V = \frac{g\,(1 - e^{-Kt})}{K}$$

Problem 2.11: A particle moving with uniform retardation covers three successive equal distances. The average velocity during the first an the third parts of the journey being 20m/s and 12m/s respectively. Determine the average velocity in the middle part of the journey.

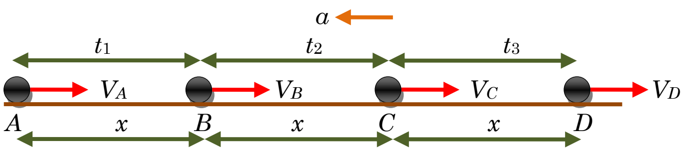


Diagram 2.9 Problem 2.11

Solution: From A to B,
$$20 \text{ m/s} = \frac{x}{t_1} = \frac{V_A + V_B}{2}$$

From B to C,
$$z = \frac{x}{t_2} = \frac{V_B + V_C}{2}$$

From C to D,
$$12 \text{ m/s} = \frac{x}{t_3} = \frac{V_C + V_D}{2}$$

The overall average velocity from A to D will be
$$\frac{V_A + V_D}{2} = \frac{3x}{t_1 + t_2 + t_3}$$

or,
$$20 + 12 - z = \frac{3x}{t_1 + t_2 + t_3}$$

or,
$$32 - z = \frac{3x}{\frac{x}{20} + \frac{1}{z} + \frac{x}{12}}$$

$$z = [\sqrt{(241)} + 1] \text{ m/sec.}$$

Problem 2.12: The Diagram 2.10a shows a car and a train moving initially with velocity $V/2$ and V respectively. The car and the train accelerate at $2a$ and a respectively. The maximum speed of the car is $3V$ and that of the train is $2V$. It is also given that the length of the train is l, and the size of the car is negligible.
If $l > 3V^2/16a$, show that the car overtakes the train in time.
$l/V + 17V/16a$; and the distance travelled by the train during this time is $2l + 13V^2/8a$.

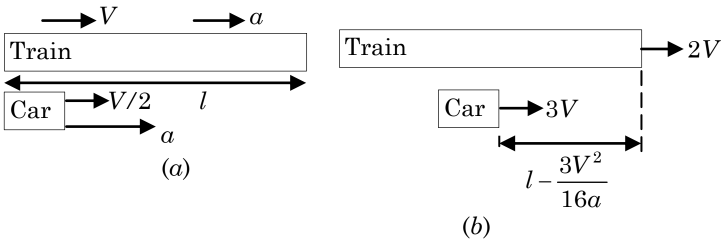


Diagram 2.10 Problem 2.12

Solution: The car and train move under constant acceleration forever. Since the acceleration of car is greater than the train, therefore, the car will overtake to the train. In the frame of reference of the train, the initial velocity of car will be $-V/2$, and acceleration of car will be

$$l = -\frac{V}{2}t + \frac{1}{2}at^2$$

If the train and car finally get the same constant maximum velocity than the car will be overtake to the train. If the maximum velocity of car is greater than the maximum velocity of train, the car may have the lesser acceleration as compared to train.
The car will overtake the train if it travels at a distance $x + l$ in the same time. In which the train travels a distance x.
Time taken by the train and car to get the maximum velocity

For train, $$t_1 = \frac{2V - V}{a} = \frac{V}{a}$$

For car, $$t_2 = \frac{3V - V/2}{2a} = \frac{5V}{4a}$$

Now, let us check the car cross the train if it attains its maximum velocity.

By train, $$s_1 = Vt_1 + \frac{1}{2}at_1^2 + 2V(t_1 - t_2)$$

or, $$= V\frac{V}{a} + \frac{a}{2}\left(\frac{V}{a}\right)^2 + 2V\left(\frac{5V}{4a} - \frac{V}{a}\right)$$

or, $$s_1 = \frac{2V^2}{a}$$

By car, $$s_2 = \frac{V}{2}t_2 + \frac{1}{2}2at_2^2$$

or, $$s_2 = \frac{35V^2}{16a}$$

Thus, $$s_2 - s_1 = \frac{3V^2}{16a}$$

Therefore, $l > \dfrac{3V^2}{16a}$

According to inequality we find that the car does not cross the train in time t_2.

In the frame of reference of the train the car has to travel on a distance $l - \dfrac{3V^2}{16a}$ to cross the train.

Now, $t_3 = \dfrac{1}{V}\left(l - \dfrac{3V^2}{16a}\right) = \dfrac{l}{V} - \dfrac{3V}{16a}$

Thus, total time $t = t_2 + t_3$.

or, $= \dfrac{5V}{4a} + \dfrac{l}{V} - \dfrac{3V}{16a}$

or, $t = \dfrac{l}{V} - \dfrac{17V}{16a}$

Problem 2.13: A body is moving under the action of multiple velocity vector given by the following relations

$$V = 2i + 3j + 5k \quad 0 \le t \le 2\text{sec}$$
$$V = -1i + 3j + 0k \quad 2\text{sec} \le t \le 4\text{sec}$$

Find out the overall displacement vector if initially the body was at the origin. What is the overall average velocity vector?

Solution: Now,

$$\begin{matrix} 0 & 0 & 0 \\ 4 & 6 & 10 \\ -2 & 6 & 0 \end{matrix}$$

After solving, we get $2i + 12j + 10k$
where $2i + 12j + 10k$ can be called the displacement vector, or, position vector.

Problem 2.14: A body is initially on the position vector $R = 2j$ and after sometime the particle reaches the point (2, 2) moving on a circular path in the xy plane. If the radius of the circle is 5m and the particle moves in clockwise-sense on the shorter path at a constant speed 1 m/s, find out the average acceleration.

Solution: Now initial and final velocity is

$$V_i = V\cos\theta\, i + V\sin\theta\, j,$$
$$V_f = V\cos\theta\, i - V\sin\theta\, j,$$

The magnitude of the difference in the velocity is

$$|V_f - V_i| = 2V\sin\theta.$$

In above equation first term gives the change in speed, and second term gives the change in velocity.

$$\text{Arc } PQ = 5 \times 2\theta.$$

Time $= 10\theta/V$.

$$a_{avg} = \frac{V_f - V_i}{\text{time}} = -\frac{2V\sin\theta}{10\theta/V}\, j$$

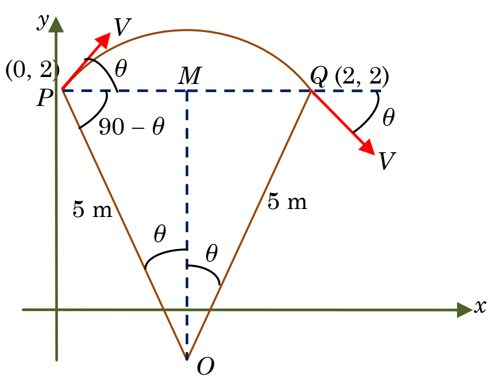


Diagram 2.11 Problem 2.14

If the point $t = 0$ is in the x-z plane then a_{avg} will be

$$= -\frac{2V\sin\theta}{10\theta / V}j$$

If the line OM makes an angle 45° with x-z plane then find the a_{avg}.

Problem 2.15: The velocity vector of a particle is a function of the displacement according to the relation $V = x^2$. (where V is in m/s & x is in metres).
Initially, the particle is at $x = 1$ m and it moves in the positive x-direction.
Find out the velocity and acceleration as function of time.

Solution: Now, it is given that $V = x^2$, then acceleration is

$$a = V\frac{dV}{dt} = x^2 \cdot 2x = 2x^3$$

Now, it is given that starting position is $x = 1$ m.
If starting point is $x = 0$, then the velocity, and, acceleration is zero. This, means that particle can never leave its position ($x = 0$), and not it gain velocity.
If $V = t^2$, $a = 2t$, then we cannot say that the body will permanently remains in the state of rest.

Now,
$$\frac{dx}{dt} = x^2$$

or,
$$\int_0^x \frac{dx}{x^2} = \int_0^t dt$$

On integration, we get
$$x = \frac{1}{1-t}$$

From the result we can say the when $t = 1$ sec, then the body reaches infinitive.

$$a = \frac{2}{(1-t)^3}$$

Problem 2.16: A particle moving in the positive x-direction has initial velocity V_0. The body is moving under retardation KV^2, where V is the instantaneous velocity vector.
Show that the velocity of the body as function of time is $V = V_0 / (1 + KV_0 t)$

Solution: Now,

$$-\frac{dV}{dt} = KV^2$$

or,
$$-\int_{V_0}^{V} \frac{dV}{V^2} = \int_0^t K\,dt$$

or,
$$\frac{1}{V} - \frac{1}{V_0} = Kt$$

After solving, we get
$$V = \frac{V_0}{V_0 Kt + 1}$$

Problem 2.17: A particle is moving on the parabola $y = x^2$ at a constant speed 1 m/s. Represent the x and y coordinates of the as function of time.

Also find out the acceleration vector of the particle. Explain the cause of the acceleration.

Solution: Now, given relation $y = x^2$,

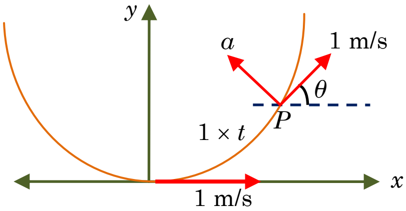


Diagram 2.12 Problem 2.17

Now, $$dy = 2x.dx,$$

Then, $$l = \int \sqrt{dx^2 + dy^2}$$

or, $$dt = \int \sqrt{dx^2 + 4x^2 dx^2}$$

or, $$dt = \sqrt{1 + 4x^2}\ dx$$

or, $$\frac{dx}{dt} = \frac{1}{\sqrt{1 + 4x^2}}$$

Now, $$\frac{dy}{dt} = \sqrt{1 - \left(\frac{dx}{dt}\right)^2}$$

or, $$\frac{dy}{dt} = \frac{2x}{\sqrt{1 + 4x^2}}$$

The acceleration vector must be perpendicular to the velocity vector.

Now, $$\frac{dy}{dx} = 2x\ , \quad \frac{d^2y}{dx^2} = 2$$

Then, radius of curvature is $$R = \frac{[1 + 2x^2]^{3/2}}{2}$$

In the given problem, the radius of curvature is minimum at $x = 0$, therefore the normal acceleration will have the maximum value of at this point.

$$a_{\max} = \frac{V^2}{R_{\min}} = \frac{1}{1/2}$$

Then, we get $a_{\max} = 2$ m/sec^2.

When $x \to \infty$, then the normal acceleration or acceleration of body becomes zero.

Problem 2.18: The radial distance of a point measured from the origin is given by the relation $R = 2t + 1$ and the angular position of the particle with respect to the x axis (the angle is being measured in the anticlockwise sense) is given by the relation $\theta = t$ (where the angle is measured in radians).

Find out the initial speed of the particle at time $t = 0$ sec.

Find out the velocity vector of the particle as a function of time.

Solution: Given relation is $R = 2t + 1$, and $\theta = t$.

If θ is constant and R is changing then the motion has to be straight line. If R is constant and θ is changing then the motion has to be circular.

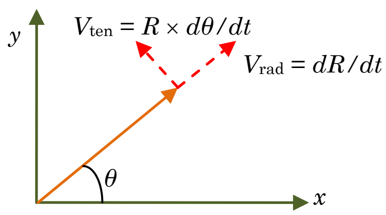


Diagram 2.13 Problem 2.18

$$V_{\text{rad}} = 2,\ V_{\text{ten}} = R \times d\theta/dt = 2t + 1$$

Now, speed

$$\left[V_{\text{ten}}^2 + V_{\text{rad}}^2\right]^{1/2} = [4 + 4t^2 + 1 + 4t]^{1/2}$$

Speed = √5 m/s.
This motion takes place in *x-y* plane only.

Problem 2.19: If V_1, V_2 & V_3 be the average velocities in three successive time intervals t_1, t_2 & t_3 of a point moving in a straight line with uniform acceleration then show that

$$(V_1 - V_2)/(V_2 - V_3) = (t_1 + t_2)/(t_3 + t_2)$$

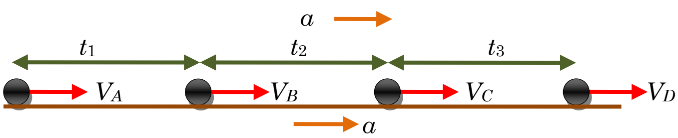


Diagram 2.14 Problem 2.19

Solution:

$$\frac{V_1 + V_2}{V_2 + V_3} = \frac{V_2 - V_1}{V_3 - V_2}$$

or,
$$\frac{(V_2 - V_1)/2}{(V_3 - V_2)/2} = \frac{V_C - V_A}{V_D + V_B} = \frac{t_1 + t_2}{t_2 + t_3}$$

Problem 2.20: If a point moving under uniform acceleration describes successive equal distances in the times t_1, t_2 & t_3 then show that

$$1/t_1 - 1/t_2 + 1/t_3 = 3/(t_1 + t_2 + t_3)$$

Solution:

$$\frac{x}{t_1} - \frac{x}{t_2} + \frac{x}{t_3} = \frac{3x}{t_1 + t_2 + t_3}$$

or,
$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$

Now, we see that LHS is equal RHS, both gives average velocity.

Problem 2.21: Prove that for a particle moving under uniform acceleration a in a straight line,

$$a = 2(x/T - y/t)/(T + t),$$

where y is the distance travelled in the t seconds and x is the distance travelled in the next T seconds.

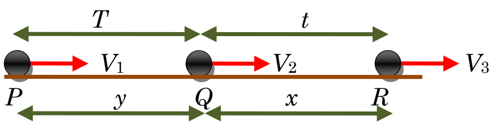


Diagram 2.15 Problem 2.21

Solution:

$$a = \frac{2\left[\dfrac{V_3 + V_2}{2} - \dfrac{V_1 + V_2}{2}\right]}{T + t}$$

or,
$$a = \frac{V_3 - V_1}{T + t}$$