Problems of Practices
Of
Mechanics
Chapter-9 Vertical Circular Motion

Prepared By

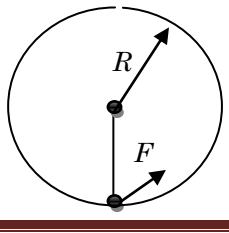
Brij Bhooshan
Asst. Professor
B. S. A. College of Engg. And Technology
Mathura, Uttar Pradesh, (India)

Supported By:
Purvi Bhooshan

Please welcome for any correction or misprint in the entire manuscript and your valuable suggestions kindly mail us brijrbedu@gmail.com.

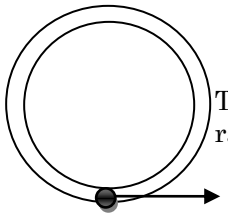
PROBLEMS ON VERTICAL CIRCULAR MOTION

1. A particle slides from rest at a depth $R/2$ below the highest point down the outside of a smooth sphere of radius R. Prove that it leaves the sphere at a height $R/3$ above the Center. (You can make reasonable assumptions.)
2. A particle of mass M attached to a massless rigid describes a vertical circular motion The speeds at the lowest and the highest positions are $3U$ and U respectively, prove that the tension in the rod when its angle with the downward vertical is θ, is equal to $Mg(3\cos\theta + 2.5)$.
3. A particle is connected to rigid rod in a zero gravity region. Initially the particle is in rest at the position shown. A constant force F has been applied on the particle. Find out the maximum velocity of the particle and also find out the acceleration at that position. The initial angle made by the force vector with the radius is θ.



4. A heavy particle of mass M, oscillates through an angle 180° on the inside of a smooth circular hoop of radius R fixed in a vertical plane. Prove that the normal reaction on the hoop at any point is $3MV^2/2R$, where V is the velocity at that point.

5. Inside the spherical tube shown, a small particle of mass M is oscillating through an angle 180°. When the particle is at its lowest position, the tube is given a constant horizontal acceleration of $2g$. Find out the angle on which the mass will oscillate. Radius is R. No friction.

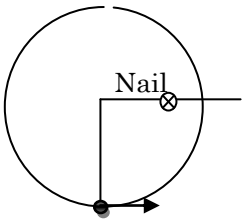


6. A mass m hangs at one end of a string of length L, the other end is fixed. The mass is given a horizontal velocity $[7gL/2]^{1/2}$. Show that the tension in the string becomes zero when the string makes an angle 60° with upward vertical. Also show that tension in the string at that position is $3mg$.

7. A particle hanging from a fixed point by a string of length L, is projected horizontally with speed $[gL]^{1/2}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of motion when the tension in the string is equal to its weight.

(Answer: $[gL/3]^{1/2}$)

8. A particle connected to a string of length L, has velocity $[6gL]$ at the lowest position. The tension at the highest position is T_1. Now, nail is fixed at a distance X from the center on the horizontal diameter. The radius of the new circle is $L–X$. At the highest point of the new circle let the tension be T_2.
If $T_2/T_1 = 3$ then find out the value of X.



9. A particle slides from the top down the outside smooth surface of a fixed sphere of radius R. Find the initial horizontal velocity that must be given so that the particle leaves the surface at a point, whose vertical height above the center of the sphere is $3R/4$.

(Answer: $[gR]^{1/2}/2$)

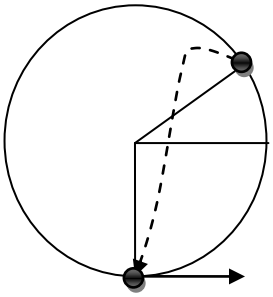
10. A small body is placed on the top of a smooth sphere of radius R. The sphere is given a constant horizontal acceleration A. The body starts sliding on the sphere. When ϕ is the angle between the radius vector and the vertical, the particle looses contact. Show that ϕ can be calculated by the help of the equation below

$$\cos\phi - (A/g)\sin\phi = 2/3$$

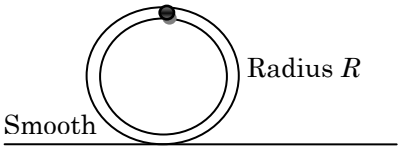
Also show that the velocity of the body relative to the sphere at that instant will be equal to $[2gR/3]^{1/2}$.

11. A particle connected by a string of length L is given a horizontal velocity V. The string slackens at some point on the upper half of the circle. The particle moves

on a parabola and passes through its initial position. Find the initial velocity given to the particle. (Answer: $[7gL/2]^{1/2}$)

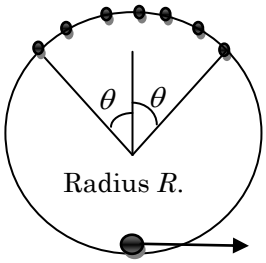


12. Inside a smooth spherical tube, a particle of mass m has been placed at the highest position. The mass of tube is M. The particle is given a very gentle push at the highest position. Find out the velocity of the particle and the tube with respect to earth when the particle is at its lowest position.

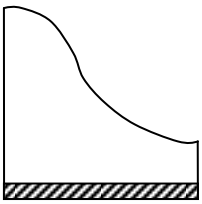


13. A small particle is placed inside circular surface, which is incomplete at the top, a shown. What should be the velocity of the particle at the lowest point so that it can join the circular surface after leaving it?

(Answer: $[gR(2 + 2\cos\theta + \sec\theta)]^{1/2}$)

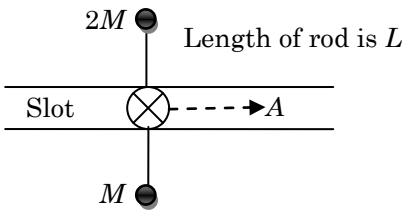


14. In the diagram shown, the small ring can slide on the circular rods fixed on the block. The mass of the ring is equal to the mass of the block. The ring is given a very small velocity with respect to the rod. Find out the velocity of the ring during its motion at the point where the centrifugal force acting on it is zero.



The circular rod is made up of two quarter circles. Each of radius R.
Friction is completely absent.

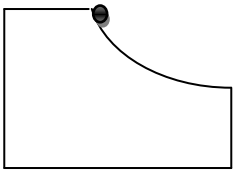
15. Two point masses of M and $3M$ have been fixed at the ends of rigid rod which can rotate about the point O. Assume gravity to be absent and the rod to be rigid and massless. The center O of the rod is constrained to move inside a slot. The center is being accelerated with A inside the horizontal slot. Find out the maximum angular velocity of the rod during the motion.



16. In the previous question, let the rod have an angular displacement of θ. Find out the normal reaction acting at the center O as a function of θ. If the whole system is moving under constant acceleration A then find out the force applied by the external agent at the center O as function of θ.

17. On the smooth quarter circular surface of the block, a particle is released from the state of rest. The coefficient of friction between the block and the ground is μ. Show that the condition for the block to not slip on the ground as the particle slides on it is as below

$$\mu > \frac{\left[(1+2K)^2 - 1\right]^{1/2}}{4K(1+K)}, \text{ where } K = M/3m.$$



The block has mass M and for particle has mass m.

18. A large ring of radius R passes inside a small ring. The large ring is vertical and the coefficient of friction between the large and the small ring is equal to μ. The large ring is rotating under constant angular velocity ω. Show that if the following inequality holds true then the smaller ring will not slip on the larger ring

$$\omega > \left[g/R\right]^{1/2}\left[1+\mu^{-2}\right]^{1/4}$$

19. A bead ("MOTI") can slide freely on a fixed smooth circular arc. The bead has been connected by a string, which passes over a fixed pulley. A constant force F is pulling the string.

a) Find out the velocity of the bead when it reaches the highest position.

b) Find out the initial acceleration of the bead.