Problems of Practices
Of
Mechanics
Chapter-4 Projectile Motion

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PROBLEMS ON PROJECTILE MOTION

1. If for a projectile V is the speed of projection, R is the horizontal range, T is the time of flight and H is the maximum height, prove the following relations
   a) $g^2T^4 - 4T^2V^2 + 4R^2 = 0$.
   b) $16gH^2 - 8V^2H + gR^2 = 0$.
   c) $V^2 = 2g[H + R^2/16H]$
2. A particle is thrown with a velocity U in a manner such that the horizontal range is double the maximum height; prove that its horizontal range is $4U^2/5g$.
3. A particle is projected with initial velocity U at an angle θ with the horizontal. After how much time will the direction of velocity vector become perpendicular to the initial velocity vector?
4. Two projectiles are projected from the same point at the same time with initial velocity u and v and at angle α and β respectively. Prove that after time $t = uv \sin(\alpha - \beta) / [g(v \cos\beta - u \cos\alpha)]$ their velocity vectors become parallel.
5. A particle is projected from the origin with its initial velocity components along horizontal and vertical to be as U_1 and U_2 respectively. The particle passes through the point (h, k). Prove that $2U_1^2 2k + gh^2 = 2U_1U_2h$.

6. A particle is thrown at a point, which is in the horizontal plane through the point of projection. In the first throw, the particle falls short of the target by a meters. In the first throw, the angle made by the projectile with the horizontal is α. In the second throw from the same position, the particle falls ahead of the target by b meters. This time the angle of projection β. Show that if velocity of projection is sane in all the cases then the correct angle of projection will be

$$\frac{\sin^{-1}\left[(a\sin 2\beta + b\sin 2\alpha)/(a+b)\right]}{2}$$

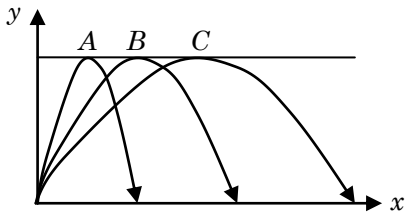
7. A projectile thrown from the origin in the XY plane, has equation of path $y = ax - bx^2$, where a and b are positive constants. If acceleration due to gravity g m/s^2 is directed in the –Y direction then find out the value of the followings by the help of equation.

a) Horizontal range
b) Maximum height.
c) Initial velocity and angle of projection.

8. A projectile is to hit a target having coordinates (a, b). The projectile is fired from the origin with a velocity $[2ga]^{1/2}$. Show that it is impossible to hit the target if $b > 3a/4$.

9. The diagram shows trajectories of three projectiles A, B and C. Arrange in the increasing order their

a) time of flight
b) initial vertical component of velocity.
c) projection speed.



Special Problems:

10. A particle is projected from the origin O at an angle α; it passes through a point P and strikes the horizontal plane at point Q. If $\angle POQ = \gamma$ and $\angle PQO = \beta$ then prove that

$$\tan\alpha = \tan\beta + \tan\gamma.$$

11. A gun is fired from a moving platform. When the platform is moving in the forward direction, the range of the gun is R. When the platform is moving in the backward direction, the range of the gun is S. In both the cases the platform is moving with a velocity of V m/s. Show that the angle of firing with respect to the platform is equal to

$$\tan^{-1}\left\{\frac{\left[g(R-S)^2\right]}{4V^2(R+S)}\right\}$$

12. A particle is projected from a point at a height $3H$ above horizontal plane. The direction of projection makes an angle α with the horizon. Show that if the greatest height above the point of projection is H, then the horizontal distance travelled before striking the plane is $6H\cot\alpha$.

13. T_1 and T_2 are two times of flight with which a given range R on the horizontal plane can be reached by a particle projected with velocity U. Prove that T_1 and T_2 satisfy the equation

$$g^2T^4 - 4T^2U^2 + 4R^2 = 0$$

Questions of Collision of Two Projectiles:

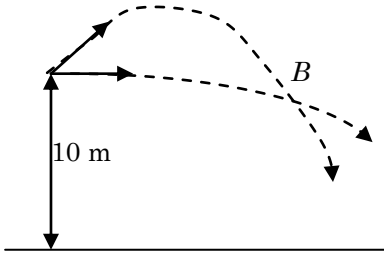
14. Two projectiles A and B are thrown from the points (0, 0, 0) and (10, 0, 0) respectively. The initial velocity vectors are

$$V_A = 1i + 1j + 1k \;\&\; V_B = ai + bj + ck$$

The acceleration due to gravity is in the Y direction. Find out the value of a, b and c if the two projectiles collide in mid air. (Do not use the relative motion techniques. Solve by general method.)

15. A projectile is thrown from point A at time $t = 0$ as shown. After some time T, another particle is thrown horizontally with the same projection speed. The two projectiles collide at point B. Find out the value of T.

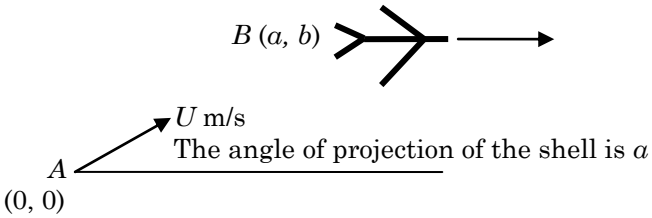
(Answer 10/g seconds)



The projection speed is $5\sqrt{3}$ m/s
The angle of projection of the 1st projectile is 60°.

16. The aeroplane is flying horizontally with velocity V. At the instant when it is at point B, a shell is fired from A. Show that the condition for the shell to hit the plane is

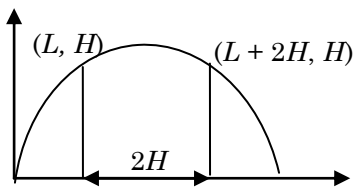
$$b = \frac{Ua\sin\alpha}{(U\cos\alpha - V)} - \frac{ga^2}{2(U\cos\alpha - V)^2}$$



Challenge Problems:

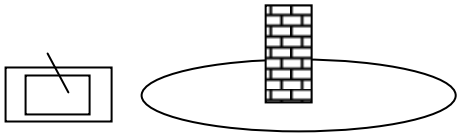
17. A body is projected at an angle α to the horizon so as to just clear two walls of equal height H at a distance $2H$ from each other. Show that the range is equal to $2H \cot(\alpha/2)$.

Hint: In this question, L is an unknown. Assume the eqn. of the parabola to be path $y = ax - bx^2$, wt $a = \tan\alpha$ & b/a= range. On putting $y = H$, the difference of the roots of the eqn. should be $2H$.



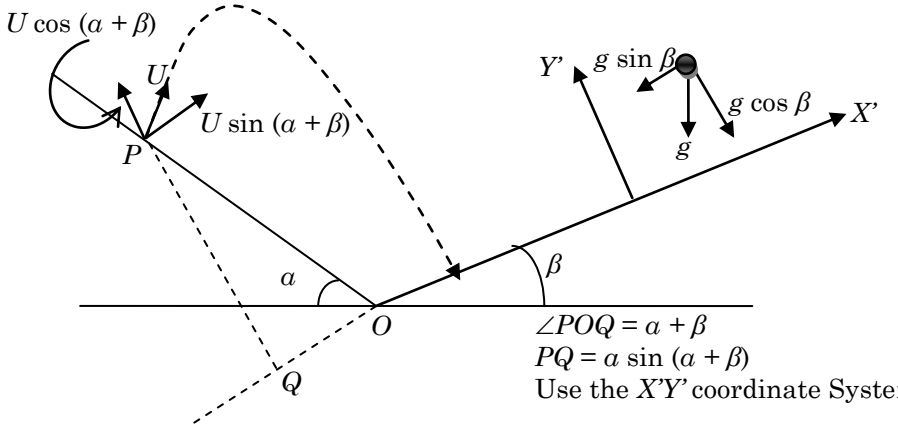
18. A regular hexagon stands up with one side on the ground and a particle is projected so as to graze its four vertices. Show that the velocity of the particle on reaching the ground is to the least velocity as $\sqrt{31}$ to $\sqrt{3}$,

19. A gun is placed on the top of a tower of height H. The tower is surrounded by sea. In the sea, a ship is present having a gun similar to the gun placed on the top of the tower. Both guns have the same projection speed equal to $[2gk]^{1/2}$.
Show that there is a region of area $8\pi Hk$ in the sea in which the gun placed in the tower is out of the range of the gun present in the ship but the gun present in the ship is in the range of the gun present in the tower.



Question to be Solved by Changing the Coordinate System

20. Two inclined planes intersect each other in a horizontal plane. If a particle is projected at an angle $\pi/2$ as shown, and hits the other plan at $\pi/2$. Find out the value of U. [Given $OP = a$]



[Ans: $U = \sin\beta\,[2ag/\{\sin\alpha - \sin\beta\cos(\alpha+\beta)\}]^{1/2}$]