Problems of Practices
Of
Mechanics
Chapter-2 Motion on Straight Line

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PROBLEMS ON MOTION ON A STRAIGHT LINE

1. A body travels half of the total distance with velocity V_1 and the other half with velocity V_2. What is the average velocity of the body?

(Answer: $2\ V_1\ V_2/[V_1 + V_2]$)

2. A car accelerates from rest with at a rate A for sometime. After which it decelerates at a rate 'B' for sometime till it comes to a state of rest. If the total time of motion is T then find out the distance travelled by the car.

(Answer: $ABT^2/(2A + 2B)$) IIT–JEE 1980

3. A car moving with constant acceleration covers the distance between two points 60 m apart in 6 seconds. Its speed as it passes the second point is 15 m/s. Find out the following

a) acceleration b) velocity at the first point.

(Answer: 5/3 m/s^2; 5m/s)

4. A racing car, moving under constant acceleration, covers the first kilometre in 30 sec. and the second kilometre in 20 seconds. Find out the acceleration and speed of the car.

(Answer: 2/3 m/s^2; 70/3 m/s)

5. A particle P_1 starts moving on a straight line with speed u and acceleration a. One second later, another particle P_2 starts from the same point with speed $u/2$ and acceleration $2a$ on the same straight line. When P_2 crosses P_1, their velocities are 31 m/s and 22 m/s respectively. Find out the distance travelled by P_1.

(Answer: 48m)

6. Two cars are moving in the same direction with the same speed with separation of 40 m. In order to overtake the first car, the second car accelerates and crosses it in 5 sec. What is the acceleration of the second car?

(Answer: 3.2m/s^2)

7. A man with constant reaction time can stop his car within 30m when it is moving at a speed of 72 km/hr and within 10 m when it is moving at a speed of 36 km/hr. What is the distance within which he can stop the car if it is travelling at a speed of 54 km/hr?

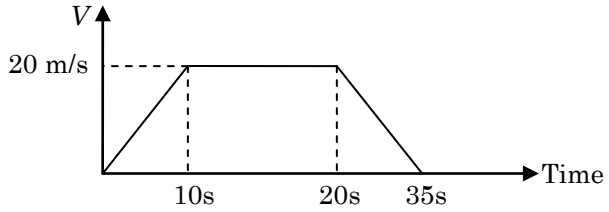
(Answer: 18.75m)

8. It takes 2minutes to acquire full speed of 60 km/hr from rest and exactly 1 minute to come to rest from full speed. If somewhere in between two stations l km of the track is being repaired and the speed limit is 20km/hr, how late will the train be due to repair works?

(Answer: 2min 40sec)

9. A particle moves on straight line and its velocity changes with time as shown.

a) Calculate the initial acceleration and the final acceleration.
b) Calculate the distance travelled by the body in the first 15seconds.
c) Draw the acceleration versus time graph.



PROBLEMS ON MOTION UNDER GRAVITY

1. A particle is thrown up with an initial velocity u. If after T_1 and T_2 time the particle is at height H, then prove that

$$H = gT_1T_2/2;\ u = g(T_1 + T_2)/2;\ \text{Maximum height} = g(T_1 + T_2)^2/8$$

2. A body is projected vertically upwards from point A, top of a tower. It reaches the ground in T_1 seconds. If it is projected vertically downwards from A with the same initial velocity, it takes T_2 seconds. Show that if we allow it to fall freely under gravity then it should take $[T_1T_2]^{1/2}$.

3. From a point A, 80m above the ground, a particle is projected vertically upward with a velocity 29.4m/s. Five seconds later, another particle is dropped from a point B, 34.3 m vertically below A. Determine when and where will the two particles meet.

(Answer: 8seconds; 1.6m above the ground)

4. A particle is projected vertically upwards, and T seconds afterwards another particle is projected upwards with the same initial velocity U. Prove that the two particles will meet after time $T/2 + U/g$. Also find the velocity of the two particles when they meet.

(Answer : $gT/2$ and $-gT/2$)

5. A particle is dropped from the top of a tower. After the particle has fallen a distance X, another particle is dropped from a distance Y below the top. If both the particles reach the ground in the same time, prove that the height of the tower is equal to $(X + Y)^2/4X$.

[is $X > Y$ or $X < Y$?]

6. A particle is projected vertically upwards. Prove that it will be at 3/4 of its maximum height at times, which are in the ratio 1:3.

7. A particle moves under a constant acceleration and it covers distance a, b end c the p^{th}, q^{th} and r^{th} respectively.

Prove that $\quad a(q - r) + b(r - p) + c(p - q) = 0.$

8. A body falls freely under gravity from O passing through point A, B and C. It is given $AB = BC$. If the particle takes 2 seconds to go from A to B and 1 sec. to go from B to C, find the distance AB.

9. A particle is thrown vertically upwards and the resistance due to air is not neglected. Assume that air provides constant retardation K m/s^2 always acting opposite to the direction of motion. Show that the time taken to rise is less than the time taken to fall.

If the initial velocity is V m/s than find out the velocity with which the particle returns back to the starting point.

CHALLENGE PROBLEMS

1. Three particles start moving from the origin at the same time. Particle 1 goes in the x- direction with velocity V_1. Particle 2 goes in the y-direction with velocity V_2. Particle 3 goes along the line $y = x$ and its velocity V_3. Show if all the three particles always remain on the same straight line in the subsequent motion, then

$$V_3 = \sqrt{2}\, V_2\, V_1 / (V_2 + V_1)$$

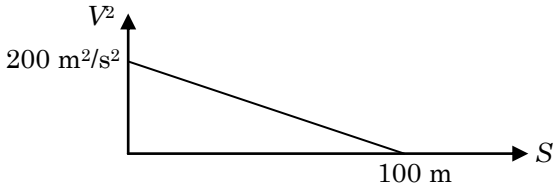
2. A particle moving with uniform retardation covers three successive equal distances. The average velocity during the first an the third parts of the journey being 20m/s and 12m/s respectively. Determine the average velocity in the middle part of the journey.

(Answer: $(\sqrt{(241)} + 1)$ m/s)

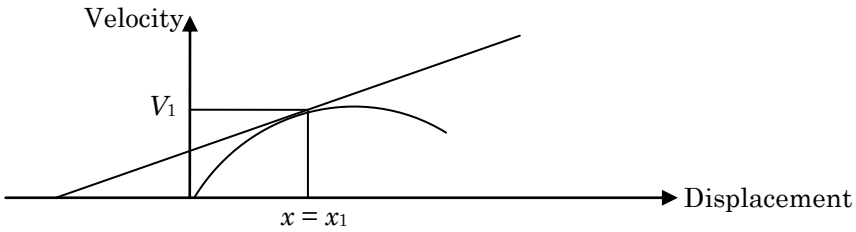
3. A truck has to carry a load in the shortest time from one point to another at a distance L. It can only start up or slow down at the same acceleration a. What is the maximum velocity with which the truck can move?

(Ans: $\sqrt{(aL)}$

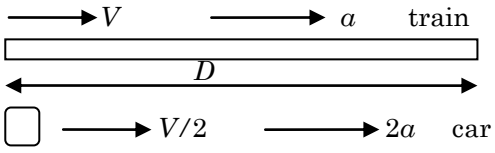
4. A body moves in a straight line with a decreasing velocity whose square is linear with distance S as shown. Find the time required for the body to travel the 100m and the distance ΔS travelled during the last 2 sec before coming to rest.



5. Using the graphical approach, find out the acceleration at $x = x_1$.



6. The figure shows a car and a train moving initially with velocity $V/2$ and V respectively. The car and the train accelerate at $2a$ and a respectively. The maximum speed of the car is $3V$ and that of the train is $2V$. It is also given that the length of the train is D, and the size of the car is negligible.
If $D > 3V^2/16a$, show that the car overtakes the train in time.
$D/V + 17V/16a$; and the distance travelled by the train during this time is $2D + 13V^2/8a$.



PROBLEMS ON VARIABLE VELOCITY VECTOR

1. The variation of the x and y coordinate of a particle moving in xy plane is as following: $x = at + bt^2$ & $y = t^2$, where a and b are constants.
Find the
   a) path of the body
   b) instantaneous velocity vector
   c) instantaneous acceleration vector.
   d) the average velocity and acceleration vector on the time interval $t = 0$ sec. to $t = 2$sec.

2. A body is moving under the action of multiple velocity vector given by the following relations

$$V = 2i + 3j + 5k \quad 0 \le t \le 2\text{sec}$$
$$V = -1i + 3j + 0k \quad 2\text{sec} \le t \le 4\text{sec}$$

Find out the overall displacement vector if initially the body was at the origin. What is the overall average velocity vector?

3. A body moves on the following velocity vectors

$$V = 3i + 2j - 7k \quad 0 \le t \le 3\text{sec}$$
$$V = 4i - 1j + 4k \quad 3\text{sec} \le t \le 5\text{sec}$$
$$V = ai + bj + ck \quad 5\text{sec} \le t \le 7\text{sec}$$

Find out the value of a, b and c, if the body at $t = 7$sec. reaches the starting point.

4. A body is initially on the position vector $R = 2j$ and after sometime the particle reaches the point (2, 2) moving on a circular path in the xy plane. If the radius of the circle is 5m and the particle moves in clockwise-sense on the shorter path at a constant speed 1 m/s, find out the average acceleration.

5. The velocity vector of a particle is a function of the displacement according to the relation $V = x^2$. (where V is in m/s & x is in metres).
Initially, the particle is at $x = 1$ m and it moves in the positive x-direction.
Find out the velocity and acceleration as function of time.

6. The displacement of a particle which moves along a straight line is given by

$$x = 4t^3 + 3t^2 - 6;$$

where x is in meters and t is seconds.
   a) Find the time taken by the particle to acquire a velocity of 6m/s from rest.
   b) Find the acceleration of the particle when its velocity is 6m/s.
   c) Find the displacement of the particle during the fifth second.

(Ans: 1/2 sec; 18 m/s^2; 271 m)

7. A particle moving in the positive x-direction has initial velocity V_0. The body is moving under retardation KV^2, where V is the instantaneous velocity vector.
Show that the velocity of the body as function of time is $V = V_0 / (1 + KV_0t)$

8. A particle starts from the state of rest at time $t = 0$ sec. and moves on a straight line. The acceleration of the particle is given by the relation $A = ae^{-bt}$, where t represents time, a and b are positive constants.
Show that the maximum velocity of the particle a/b.
Also show that the displacement measured from the position of rest to any time t is

$$S = \frac{a\left[bt - \left(1 - e^{-bt}\right)\right]}{b^2}$$

9. A particle moving in a straight has initial velocity V_0 and it is moving under constant retardation KV^n, where K is a positive constant and n is greater than unity.
Show that velocity and displacement as function of time are as following

$$V = \left[V_0^{1-n} + (n-1)kt\right]^{1/(1-n)}$$

$$S = \frac{\left[V_0^{1-n} + (n-1)kt\right]^{(n-2)/(n-1)} - V_0^{2-n}}{(n-2)k}$$

10. A particle starts moving in the XY plane according to the equations:

$$X = 2t^2 \text{ and } Y = 2t + 1$$

What is the rate of change of speed at time t = 2sec?

11. A particle is moving towards a point under the action of force $F = k/(\text{distance})^3$; the distance is measured from the point and the force is always directed towards the point.
The particle is initially at distance 'a' from the point and it is in the state of rest. Show that when the particle is at a distance 'b' from the point then its velocity is $[k(a^2 - b^2)]^{1/2}/(ab)$ and the time taken to reach the point $[a(a^2 - b^2)]^{1/2}/(ab)$.

12. A particle is moving on the parabola $Y = X^2$ at a constant speed 1 m/s. Represent the X and Y coordinates of the as function of time. Also find out the acceleration vector of the particle. Explain the cause of the acceleration.

13. A particle is moving in the XY plane such that the position vector of the particle with respect to the origin is given by the relation $R = 2ti + t^2j$. Find the angular velocity vector of the particle with respect to origin.

14. The radial distance of a point measured from the origin is given by the relation $R = 2t + 1$ and the angular position of the particle with respect to the X axis (the angle is being measured in the anticlockwise sense) is given by the relation $\theta = t$ (where the angle is measured in radians).
Find out the initial speed of the particle at time t = 0 sec.
Find out the velocity vector of the particle as a function of time.

15. In the previous problem, find out the distance travelled by the particle in the first seconds.

SPECIAL PROBLEMS OF MOTION ON STRAIGHT LINE

1. If V_1, V_2 & V_3 be the average velocities in three successive time intervals t_1, t_2 & t_3 of a point moving in a straight line with uniform acceleration then show that
$$(V_1 - V_2)/(V_2 - V_3) = (t_1 + t_2)/(t_3 + t_2)$$
2. Two trains on the same line are approaching one another with velocities U and V respectively. When the distance between the trains is X, the trains see each other.
Prove that it will just be possible to avoid a collision if
$$U^2A_2 + V^2A_1 = 2 \times A_1 A_2$$
where A_1 & A_1 are the respective retardations of the trains.
3. If a point moving under uniform acceleration describes successive equal distances in the times t_1, t_2 & t_3 then show that
$$1/t_1 - 1/t_2 + 1/t_3 = 3/(t_1 + t_2 + t_3)$$
4. Prove that for a particle moving under uniform acceleration A in a straight line,
$$A = 2(X/T - Y/t)/(T + t),$$
where Y is the distance travelled in the t seconds and X is the distance travelled in the next T seconds.
5. The velocity of a train increases at a rate A from zero to V, then remains constant for an interval and finally decreases to zero at constant rate A_1. If X is the total distance travelled then prove that the total time taken is equal to
$$X/V + (V/2)(1/A + 1/A_1)$$
6. A distance S is divided into N equal parts at the end of each pan the acceleration of the moving particle increases A/N. Show that the velocity of the particle after covering the distance becomes equal to
$[AS(3 - 1/N)]^{1/2}$, where A is the initial acceleration.
7. A body starts from rest under constant acceleration. Show that the distance travelled by the body in the $(N^2 + N + 1)^{th}$ second is equal to the distance travelled in the first N seconds plus the distance travelled in the first $(N + 1)$ seconds.

8. A body is thrown up a lift with velocity U relative to the lift. The falls back after time T. Show that the acceleration of the lift in the upward direction is equal to
$$(2U - gT)/T$$
9. From a platform moving up with a velocity U with respect to earth, a particle is thrown upwards with a velocity V with respect to earth. It is given that the platform is moving upwards with acceleration A.
    a) Find out the time when the particle falls back on the platform.
    b) Find the relative velocity at the instant when the particle falls on the platform.
    c) Find the maximum relative separation between the particle and the platform during the entire motion.

At the instant when the separation is maximum , find out the velocity of the particle and the platform with respect to earth