Problems of Practices
Of
Mechanics
Chapter-14 Angular Momentum

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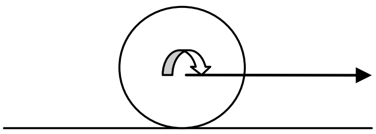
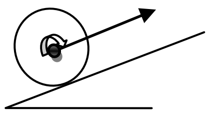
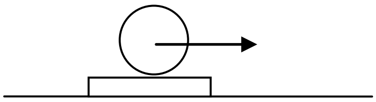
1. In diagram shown, a uniform disc of mass M is projected on a rough horizontal surface with a velocity V and angular velocity ω. Find out the linear and angular velocity of the disc when it starts rolling.



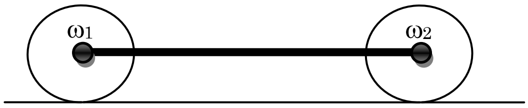
2. A uniform sphere is projected on a rough inclined plane with $V > R\omega$. The coefficient of friction is greater than $(2g \sin \theta)/7$. Find out the time after the sphere conies to a state of rest.



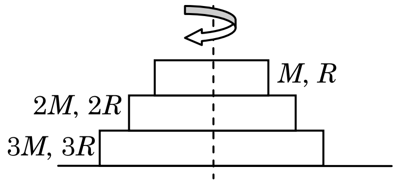
3. On the rough upper surface of a board, a disc is projected with velocity V. Find out the velocity of the board and the disc when slipping ends. Mass of the disc is M and the mass of the board is $2M$. The horizontal surface is smooth.



4. In the diagram shown both the disc have the same radius and the rod joining their centers is rigid and massless. The discs are given initial angular velocity as mentioned. The horizontal surface is rough. When the slipping stops under both the discs, find out the linear and angular velocity. Both the discs have the same mass.



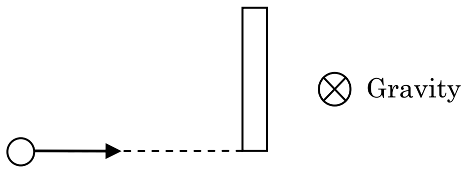
5. The diagram shows three discs having masses as mentioned. They have a common axis passing through their centers. The discs are given angular velocity ω, 2ω and 3ω in the clockwise sense from top to bottom, find out the final angular velocity when slipping between the disc end. Assume the horizontal surface to be smooth. If the horizontal surface is rough having coefficient of friction K then find out the time after all the discs come to rest. All the discs are uniform.



6. A man walks on a rough sphere so as to make it roll straight up a plane inclined at an angle θ to the horizon, always keeping himself at angle β from the highest point of the sphere. If the mass of the sphere is M and mass of the man is m then show that the acceleration of the sphere up the inclined plane is equal to

$$\frac{5g[m\sin\beta-(M+m)\sin\theta]}{7M+5m[1+\cos(\theta+\beta)]}$$

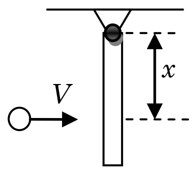
7. In the diagram shown, a uniform rod of mass M and length L is lying in the state of rest on a smooth horizontal surface. A small particle of mass m moving with a velocity V collides elastically at the end of the rod. Find out the velocity of the rod and the particle after the collision.



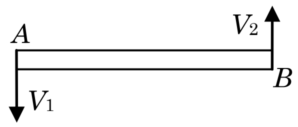
8. Repeat the above question if the collision is given to be perfectly elastic.

9. In the question number (7), suppose the particle rebounds back with velocity $V/2$ after the collision, what is the coefficient of restitution for the collision?

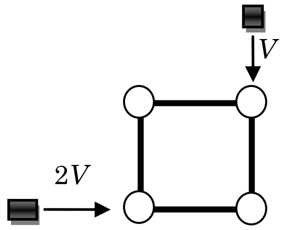
10. A uniform rod can freely rotate about the axis shown. The mass of the rod is M and its length is equal to L. A particle of mass m hits the rod elastically. If gravity is absent then find out the value of x such that force acting on the axis at the time of collision is equal to zero.



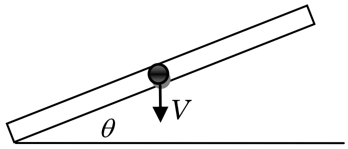
11. A uniform rod AB is falling in a vertical plane. The end A of the rod is moving downwards with velocity V_1, and the end B is moving upwards with V_2. If the end A is suddenly fixed then show that end B will rise about the point A given $V_1 < 2V_2$.



12. The diagram shows a rigid structure made up of four massless rigid rods. Point masses M have been fixed at all the vertices. The structure is placed on a smooth horizontal surface. Two bullets of mass m are moving with velocities as mentioned. They hit the point masses simultaneously and stick to the point masses. Find out the resultant linear and angular velocity of the structure. Take $M = 2m$.

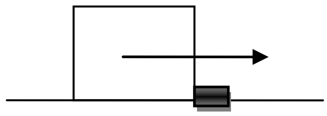


13. A rod inclined at an angle θ is dropped down such that when the rod hits the horizontal surface it is translating with velocity V. If the rod collides elastically with the floor then find out the linear and angular velocity of the rod immediately after the collision. Mass of rod of M and its length is L.



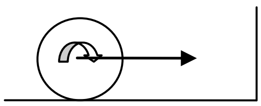
14. In the previous question, if the point of collision sticks to the ground after the collision then find out the angular velocity of the rod immediately after the collision.

15. A uniform cube of mass M and dimension L is moving on a smooth horizontal surface with velocity. Suddenly the cube meets an obstacle on the floor and after the impact the cube rotates about the obstacle. Find out the angular velocity of the cube after the collision. What should be the minimum initial velocity of the cube so that it can rotate past the obstacle?



16. In the previous question what is the amount of heat generated due to the collision?

17. On rough horizontal surface a disc is rolling with velocity V and angular velocity ω. The disc collides elastically with a smooth vertical wall. Find out the velocity off the disc when it again starts to roll.

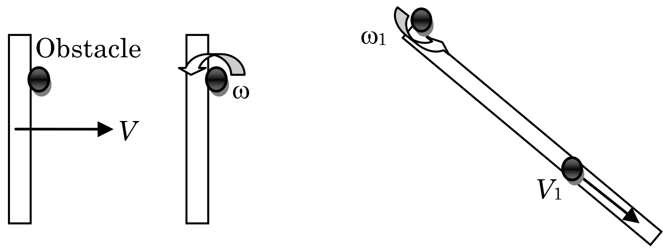


18. A perfectly rough inelastic sphere off the radius R, is rolling with a velocity V on a horizontal plane when it collides with an obstacle of height H. Show that the sphere would roll over the obstacle if

$$\frac{R\sqrt{70gH}}{7R-5H} < V < \frac{7R\sqrt{g(R-H)}}{7R-5H}$$

Further prove that the on rolling on to higher plane the sphere continues to roll with a velocity equal to $V(7R - 5H)^2/49R^2$. (Take all the regular assumptions).

19. A rod translating with a velocity V hits an obstacle and starts rotating about it. The rod slides around this obstacle and finally leaves it. Find out the angular velocity of the rod when it just about to leave the obstacle. Also find the velocity of the CM of the rod at that instant. Assume the rod to be moving on a Smooth horizontal plane.

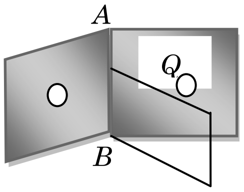


20. Two heavy metallic plates are joined together at 90° to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertial of the laminar sheet about an ends parallel to AB and passing through its centre of mass is 1.2 kg/m^{-2}. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with on angular velocity 1 rad/s and turns bade. If the impulse on the sheet due to each obstacle is 6 N-s.

(a) Find the location of the centre of mass of the laminar sheet from AB.

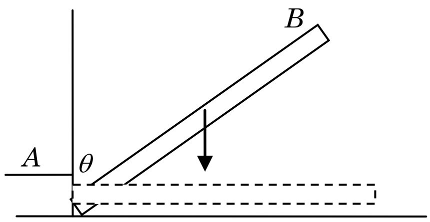
(b) At what angular velocity does the laminar sheet come back after the first impact?

(c) After how many impacts, does the laminar sheet come to rest?

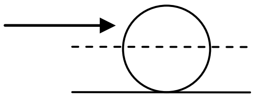


21. A uniform pole of length L is dropped at an angle θ with the vertical, and both end have a velocity u as end A hits the ground. If end A pivots about its contact point during the remainder of its motion, determine the velocity u' with which end B hits the ground. Immediately after the point of collision comes to rest and the rod starts rotating about it.

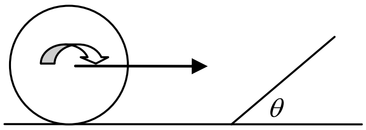
[Ans: u = $[(9u^2/4)\sin^2\theta + 3gL\cos\theta]^{1/2}$]



22. A billiard ball, initially at rest, is given a sharp impulse by a cue. The cue is horizontally a distance h above the centerline as in figure. The ball leaves the cue with a speed u and, because off its "forward english," eventually acquires a final speed off $9/7\ u$. Show that $h = 4/5R$, where R is the radius off the ball.

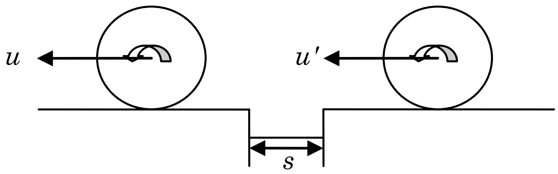


23. A uniform circular disk which rolls without slipping with a velocity u encounters an sudden change in the direction of its motion as it rolls onto the incline θ. Determine the new velocity u' of the center of the disk as it starts up the incline, and find the fraction n of the initial energy which is lost due to contact with the incline.

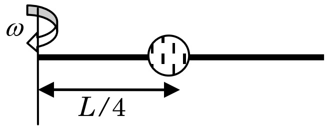


24. A uniform circular disk is rolling freely to the left with a velocity u when it encounters a depression of span s. Compute the now velocity u' of the wheel after rolling over the depression. Assume that the wheel does not slip at either edge of the depression and that it contacts the edges only and without rebound.

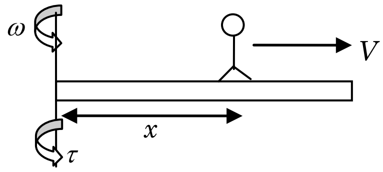
[Ans: $u' = \left[u^2\left(\dfrac{1-s^2}{3r^2}\right)^2 - 4gs^2\left(1-\sqrt{\dfrac{1-s^2}{4r^2}}\right)\dfrac{2-s^2/3r^2}{9r}\right]^{1/2}$]



25. In the diagram shown, a small bead of mass m can freely slide on a smooth rod of mass M and length L. The rod can freely rotate about the axis shown. Initially the, rod is rotating with angular velocity ω. The bead slides on the rod and reaches the end of the rod. Find out the angular velocity of the rod when the bead reaches the end. Also find out the velocity of the bead with respect to the rod when it reaches the end. Assume the bead to be in the state of rest initially with respect to the rod.

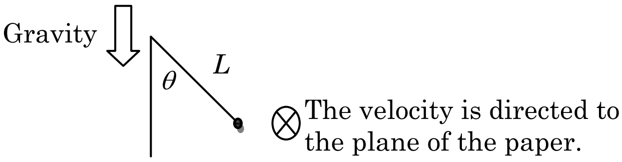


26. A rod is rotating with constant angular velocity ω and a man of mass m is walking radially outwards on the rod with a constant velocity V with respect to the rod. To keep the angular velocity of the rod constant an external must be applied. Find out the magnitude of external torque assuming the man to be a point mass. Mass of rod = M and length = L.



27. A small ball is suspended by a string of length L. The ball is shifted aside such that it makes an angle θ with the vertical. Now the ball is given a horizontal velocity V in plane perpendicular to the vertical plane in which the thread was originally located. The ball starts moving on a spiral helix. What initial velocity has to be imparted to the ball such that it deviates by a maximum angle of $\pi/2$. If at any instant in between the ball makes an $\beta(> \theta)$ with the vertical, calculate the value of velocity at this instant.

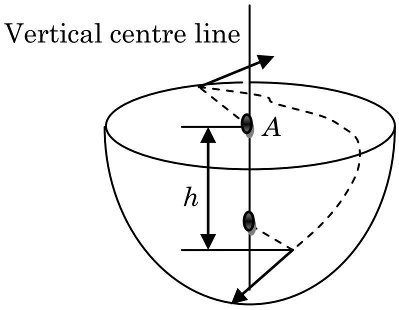
[Ans: $\sqrt{(2gL/\cos\theta)}$]



28. A small particle is given an initial velocity V_0, tangent to the horizontal rim of a smooth hollow hemisphere, at a radius r_0 from the vertical center line, as shown at point A. As the particle slides past point B, a distance h below point A and a distance r from the vertical center line, prove that the angle θ between its velocity and the horizontal tangent to the hemisphere through B is given by

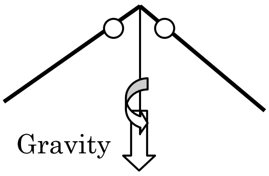
$$\cos\theta = \frac{V_0 r_0}{r\left[V_0^2 + 2gh\right]^{1/2}}$$

(Remember the particle always remain in contact hemisphere)

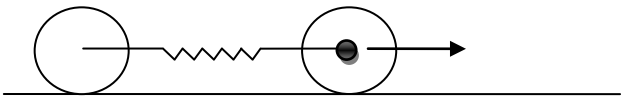


29. A heavy particle is projected horizontally along the inner surface of a fixed smooth spherical surface of radius $a/\sqrt{2}$ with a velocity $[7ag/3]^{1/2}$ at a depth $2a/3$ below the centre. Show that the particle will rise to maximum height $a/3$ above the centre. Also show that the normal reaction becomes zero at the highest point of the path.

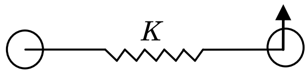
30. Identical beads are released from the state of rest. The beads can freely slide on the two identical rods. Initially the system is rotating with angular velocity ω. Find out the velocity of the beads with respect to the rods when they reach the lowest position. Both rods have length L and mass m. They are connected to each other. The mass of the bead is also m. Both the rods are inclined at an angle θ.



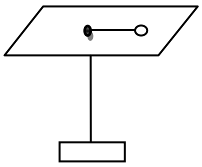
31. The diagram shows to identical spheres (solid) of mass M and radius R. Initially the front sphere has a velocity V. The spring has zero initial deformation. If the horizontal surface is perfectly rough then find out the maximum elongation in the spring. The spring is connected between the centers. The spring constant is K.



32. The diagram shows two identical particles connected by an undeformed spring of force constant K. The particles are placed on a smooth horizontal surface. If one of the masses is given a velocity V perpendicular to the length of the spring then find out the maximum elongation of the spring. The initial separation between the blocks is equal to L.



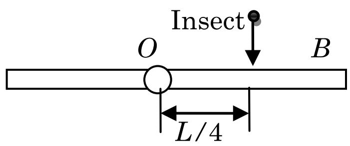
33. In the diagram shown, the particle of mass M is given a tangential velocity V and at the same time the block of mass $2M$ is released from the state of rest. No friction is present. When the block goes down a distance H, find out the velocity of both the masses.



34. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle (Neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then the tension in the string is given by $T = Ar^n$ where A is a constant, r is the instantaneous radius of the circle and n.

[Ans: $n = -3$]

35. A homogeneous rod AB of length L = 1.8m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane as shown in Figure. The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with constant angular velocity ω. (a) Determine the velocity ω in terms of V and L. (b) If the insect reaches the end B when the rod has turned through an angle of 90°, determine V.



36. A small insect moves along a uniform bar, of mass equal to itself and of length $2a$, the ends of which are constrained to remain on the circumference of a fixed circle, whose radius is $2a/\sqrt{3}$. If the insect start from the middle point of the bar and move along the bar with relative velocity V, show that the bar in time t will turn through an angle $\tan^{-1} (Vt/a) /\sqrt{3}$.

37. A heavy circular disc is revolving in a horizontal plane about its centre which is fixed. An insect, of mass $1/n^{th}$ that of the disc, walks from the centre along a radius and then flies away. Show that the final angular velocity is $n/n+2$ times the original angular velocity of the disc.

38. A uniform circular board, of mass M and radius a, is placed on a perfectly smooth horizontal plane and is free to rotate about a fixed vertical cuds through its centre; a man, of mass M', walks round the edge of the board whose upper surface is rough enough to prevent his slipping; when he has walked completely round the board to his starting point, show that the board has turned through an angle $M'4\pi/(M+2M')$.

39. In the previous question, if the man walking on the circumference were to start walking radially towards the axis then what will the angular velocity of the disc? (Solve the question assuming radial motion w.r.t. earth and w.r.t. board.)