Problems of Practices
Of
Mechanics
Chapter-13 Rotation

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S.No.	*Contents*	*Page*

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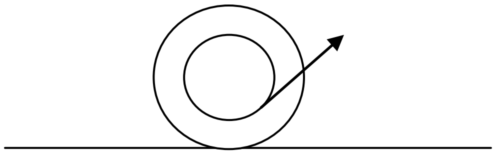
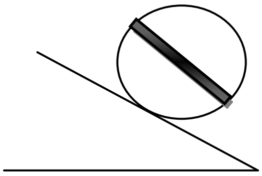
PROBLEMS ON ROLLING AND SLIPPING

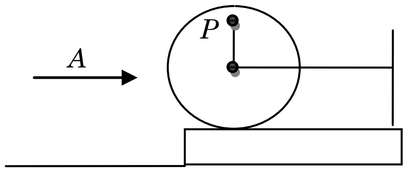
1. In the diagram shown, the spool does not slip on the horizontal plane. The thread is being pulled at an angle with the horizontal. What should he the value of the angle so that the spool can remain in the state of rest for any value of the force F? Use the details of the spool as given earlier.



2. The diagram shows a hollow spherical shell of radius R and mass M. A uniform rod is placed diametrically on the smooth inner surface of the shell. The length of the rod is $2R$ and the mass is $M/2$. The shell rolls on the inclined plane, find the acceleration of the shell down the inclined plane. The plane is inclined at angle θ. The rod is of negligible thickness.

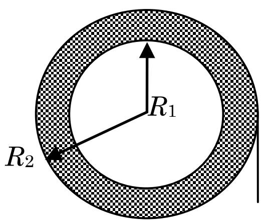


3. A non uniform disc of mass M is connected to a block by the string shown in the diagram. The center of mass of the disc is at point P. The coefficient of friction between the disc and the block is μ. The block is being accelerated with A as shown. Find out the tension in the string when the slipping is just about to start between the disc and the block. The point P is L distance above the center.

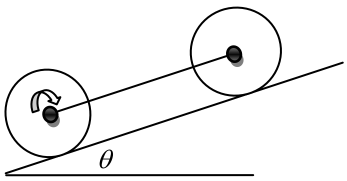


4. A tape of thickness d is wound on a disc of radius R_1. The final radius after winding the whole tape is R_2, winch is three times the original radius.
   a) Find out the total length of the tape wound on the disc. Now another tape of thickness $d/2$ is wound on the similar disc of radius R_1.
   b) If in both the cases the length of the tape is same and the winding angular velocity is also same and constant, find out the ratio of the time of winding.

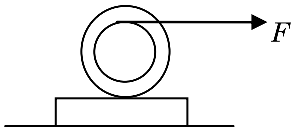
[Ans: $T_1/T_2 = 1/(\sqrt{5} - 1)$]



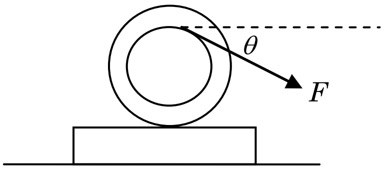
5. A uniform disc is projected on rough horizontal surface with linear velocity V and angular velocity ω. If radius of the disc is R and $V > R\omega$ then find out the final velocity.
6. A system of two identical discs connected by a massless rigid rod is present on a rough inclined plane. No slipping takes place between the discs and the plane. A torque τ is applies on the rear disc. Find out the acceleration of the whole structure up the inclined plane. The discs have mass M and radius R.



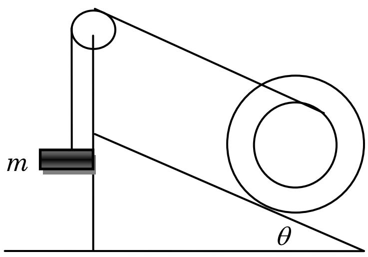
7. In the diagram shown n slipping takes place between the spool of thread and the board. The thread is being pulled by a constant force F. Using the details given in the diagram, find out the acceleration of the spool and the board. Mass of the spool is M and moment of inertia about the center is I. Mass of the board is $2M$ and the horizontal surface is smooth. Inner radius = r, and outer radius = R.



8. In the above question let the force be applied at angle θ as shown, find out the minimum value of coefficient of friction between the spool and the board such no slipping takes place.



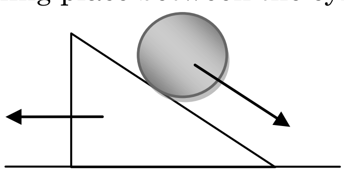
9. In the diagram shown, the spool of thread does not slip on the inclined plane. Find out the acceleration of the spool and the block. Use the details of the spool as mentioned earlier. No friction between the block and the plane.



10. A rough wedge, of mass M and inclination α, is free to move on a smooth horizontal plane; on the inclined face is placed a uniform cylinder, of mass m; show that the acceleration of the center of the cylinder down the face, and relative to it, is

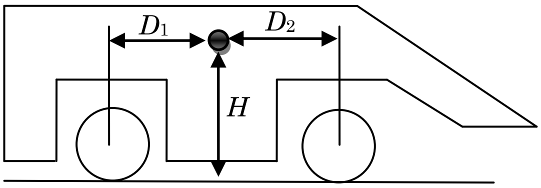
$$A = \frac{2g\sin\alpha(M+m)}{3M+m+2m\sin^2\alpha}$$

Assume no slipping to be taking place between the cylinder and the wedge.

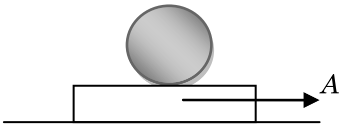


11. Three uniform spheres, each of radius R and mass M, attract each other according to the law of inverse square of distance. Initially they are placed on a rough horizontal surface with their centers forming an equilateral triangle of dimension $4R$. Show that the velocity of their centers when they collide is equal to $[G5M/(14R)]^{1/2}$, where G is universal gravitation constant.

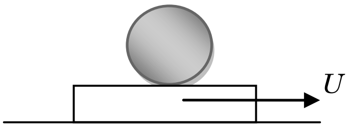
12. A motor car is driven and braked by the back wheels. The center of gravity is at a height H above the ground. The front and the back axles are respectively at distances D_2 and D_1, from the center of gravity. The coefficient of friction between the wheels and the road is equal to μ. Find out maximum acceleration is $\mu g D_2/(D_1 + D_2 - \mu H)$. Also prove that the maximum retardation is equal to $\mu g D_2/(D_1 + D_2 + \mu H)$.



13. A uniform disc has been placed on a board. If the board is moved horizontally with acceleration A and no slipping takes place between the disc; and the bond ten find out the acceleration of the disc with respect to the board.

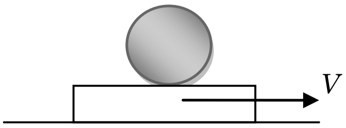


14. A uniform sphere, of mass M, rests on a rough plank (board) of mass m, which rests on a rough horizontal plane. The plank is suddenly set in motion with velocity U in the direction of its length. Show that the sphere will first slide, and then roll, on the plank, and that the whole system will come to rest in time $mU/\mu g(M + m)$, where μ is the coefficient of friction at each of the points of contact.

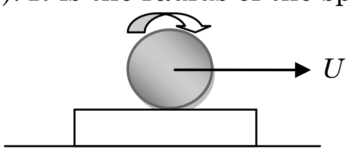


15. A board, of mass M, whose upper surface is rough and under surface smooth, rests on a smooth horizontal plane. A sphere of mass m is placed on the board and the board is suddenly given a velocity V in the direction of its length. Show that the sphere will begin to roll after a time

$$T = \frac{V}{\left(\frac{7}{2} + \frac{m}{M}\right)\mu g}$$



16. On a smooth table there is placed a board, of mass M, whose upper surface is rough and whose lower surface is smooth. Along the upper surface of the board is projected a uniform sphere of mass m. If the velocity of projection be U and the initial angular velocity of the sphere be ω about a horizontal axis perpendicular to the initial direction of projection, show that the motion will become uniform after time $2M(U - R\omega)/[(7M+2m)\mu g]$, and that the velocity of the board will then be $2m(U - R\omega)/(7M+2m)$. R is the radius of the sphere.



17. A sphere is projected with an underhand twist down a rough inclined plane; show that it will turn back in the course of its motion is $2a\omega(\mu - \tan\alpha) > 5u\mu$, where u,

ω are the initial linear and angular velocities of the sphere, μ is the coefficient of friction, and α is the inclination of the plane.

18. A sphere, of radius a, is projected up an inclined plane with velocity V and angular velocity ω in the sense which would cause it to roll up; if $V > a\omega$ and the coefficient of friction > 2/7 tan α, show that the sphere will cease to ascend at the end of a time $(5V + 2a\omega)/5g \sin \alpha$, where α is the inclination of the plane.

19. If a sphere be projected up an inclined plane, for which $\mu = (\tan \alpha)/7$, with velocity V and an initial angular velocity ω (in the direction in which it would roll up), and if $V > a\omega$, show that friction acts downwards at first, and upwards afterwards, and prove that the whole time during which the sphere rises is $(17V + 4a\omega)/18g \sin \alpha$.

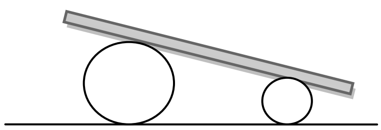
20. A hoop is projected with velocity V down a plane of inclination α, the coefficient of friction being μ(> tan α). It has initially such a backward spin ω, that after a time t_1 it starts moving uphill and continues to do so for a time t_2 after which it once more descends. The motion being in a vertical plane at right angles to the given inclined plane, show that $(t_1 + t_2)\, g \sin \alpha = a\omega - V$.

21. A uniform sphere, of radius a, is rotating about a horizontal diameter with angular velocity ω and is gently placed on a rough plane which is inclined at an angle α to the horizontal, the sense of the rotation being such as to tend to cause the sphere to move up the plane along the line of greatest slope. Show that, if the coefficient of friction be tan α, the centre of the sphere will remain at rest for a time $2a\omega/5g \sin \alpha$, and will then move downwards with acceleration $5/7g \sin \alpha$. If the body be a thin circular hoop instead of a sphere, show that the time is $a\omega/g \sin \alpha$ and the acceleration $0.5\, g \sin \alpha$.

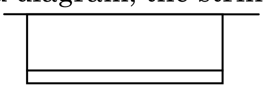
22. A uniform ring, of radius a, is propelled forward on a rough horizontal table with a linear velocity u and a backward spin ω, which is > u/a. Find the motion and show that the ring will return to the point of projection in time $(u + a\omega)^2/4\mu g(a\omega - u)$, where μ is the coefficient of friction. What happens if $u > a\omega$?

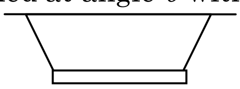
23. A rod forming an angle with the horizontal equal to θ rests on two massless rollers of different radii. Determine the acceleration of the rod if it does not slip on the rollers. Assume slipping between the rollers and the plane to also be absent.

[Ans: $g \sin(\theta/2)$]



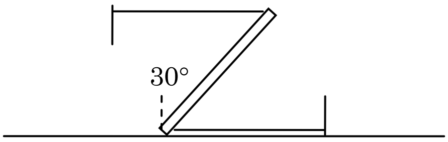
24. In the diagram shown, the rod is horizontal and balanced by two strings. The rod has length L and mass M. At time $t = 0$ sec, the string on the LHS is cut. Find the tension in the RHS string just after the string is cut. Solve for both the diagrams. In the second diagram, the strings are inclined at angle θ with the vertical.



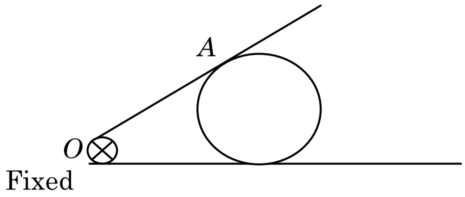


25. A uniform rod is held in the state of rest by two strings. If the lower string is cut then the acceleration of lowermost point of the rod just after the string is cut. Assume friction to be absent.

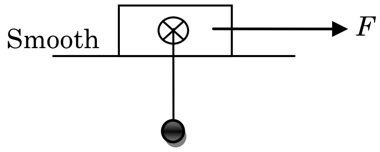
[Answer: $3g\sqrt{3} / 8$]



26. In the diagram shown, the whole system is released from the state of rest. No friction present between the rod and the cylinder. The cylinder does not slip on the ground. Mass of cylinder = M and radius = R. Mass of rod = $2M$ and length = $2R$. Find out the initial acceleration of the cylinder. The rod is initially inclined at angle θ, $OA = 4R/3$. Rod can freely rotate about the O.

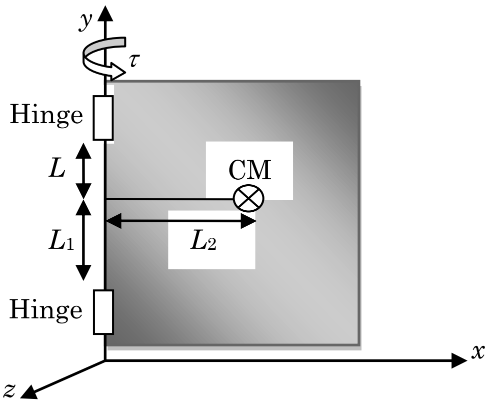


27. The block shown has mass M. A massless rigid rod is pivoted on the block. On the other side of the rod, a point mass M has been fixed. Find out the force F that should be applied on the block horizontally so that it starts moving with acceleration A. Length of the rod is L.



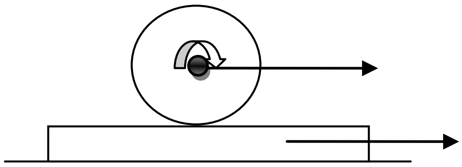
28. A homogeneous sphere, of mass M, is placed on an imperfectly rough table, and a particle, of mass m, is attached to the end of a horizontal diameter. Show that the sphere will begin to roll or slide according as u is greater or less than $5(M + m)m/ (7M^2 + 17Mm + 5m^2)$. If μ be equal to this value, show that the sphere will begin to roll.

29. The diagram shows a door hinged at two different positions. Initially the door is in the state of rest. A torque τ is applied about the axis. The moment of inertia about the axis of rotation is I and its mass is M. Find out the initial value of the contact force (or normal reaction) acting on the door from the hinges. If the door is given to have initial angular velocity ω then find out the normal reaction on the hinges in the position shown. Give your answer in vector form. Normal reaction on the upper hinge in the $+y$ direction is given to be N_1.

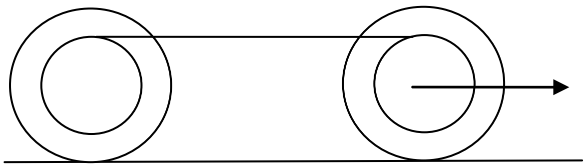


PROBLEMS ON ANGULAR KINETICS

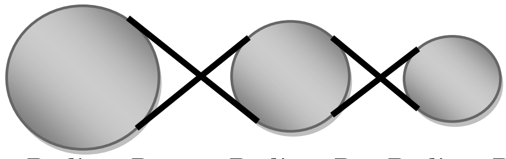
1. In the diagram shown, the board is being pulled horizontally with constant velocity Vm/s. The disc of radius R makes N revolutions in going from the rear edge of the board to the front edge. The disc is rolling on the board. Find out the displacement of the center of the disc with respect to earth.



2. In the diagram shown, both the spools of thread are identical. The center of the front spool is being moved with a velocity of Vm/s. If the no slipping takes place on the ground and the tension in the string remains non zero then find out the linear and angular velocity of the rear spool. Inner radius is r and the outer radius is R.

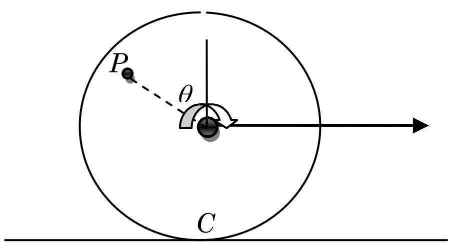


3. A belt tightly passes over the three discs. The axis of rotation of all the discs is fixed. If the first disc is rotated with angular velocity ω in the clockwise sense then find out the angular velocity of all the other discs. No slipping takes place between the discs and the belt.



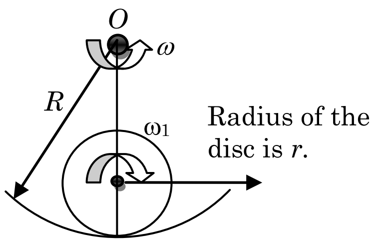
Radius: R Radius: R 2 Radius: R 3

4. A disc is rolling on a horizontal plane with linear acceleration A and angular acceleration α. At the instant when the center of the disc has linear velocity V, find out the angular velocity ω of the disc. Find out the instantaneous acceleration of the point P shown on the disc. Also find the acceleration of the lowest point C.

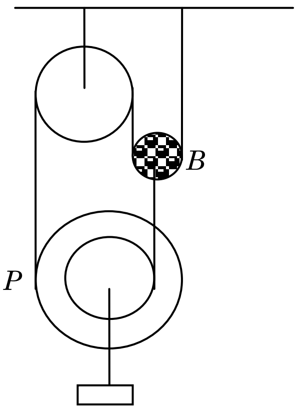


5. A disc is rolling on circular surface as shown. In the diagram, ω is the angular velocity of the center of the disc with respect to point O and ω_1 is the angular velocity of the disc with respect to the center of the disc. First of all, find out the condition of no slipping. After this find out the angular velocity of the disc with respect to O. Now prove that if V is the velocity of the center of the disc then

$$V = \left| \overrightarrow{\omega_{disc \to 0}} \right| r$$



6. The block has vertical acceleration A. On the double pulley P, string is unwinding from the RHS (that means from the radius $2R$) and then it is winding from the LHS (that means from the radius $3R$). Show that the acceleration of the pulley B will be $5A$ vertically upwards. Assume tension to be present in all the strings. Inner radius $2R$, outer radius $3R$.



7. The inner disc rolls on the inside of a hollow cylinder. The time taken by the center of the disc to complete one revolution about O is T. If the entire motion is taking place at constant speed then show that the acceleration of the point A at the instant shown, is as below.

$$A = \frac{(R_2 - R_1)(2R_1 - R_2)(2\pi / T)^2}{R_1}$$